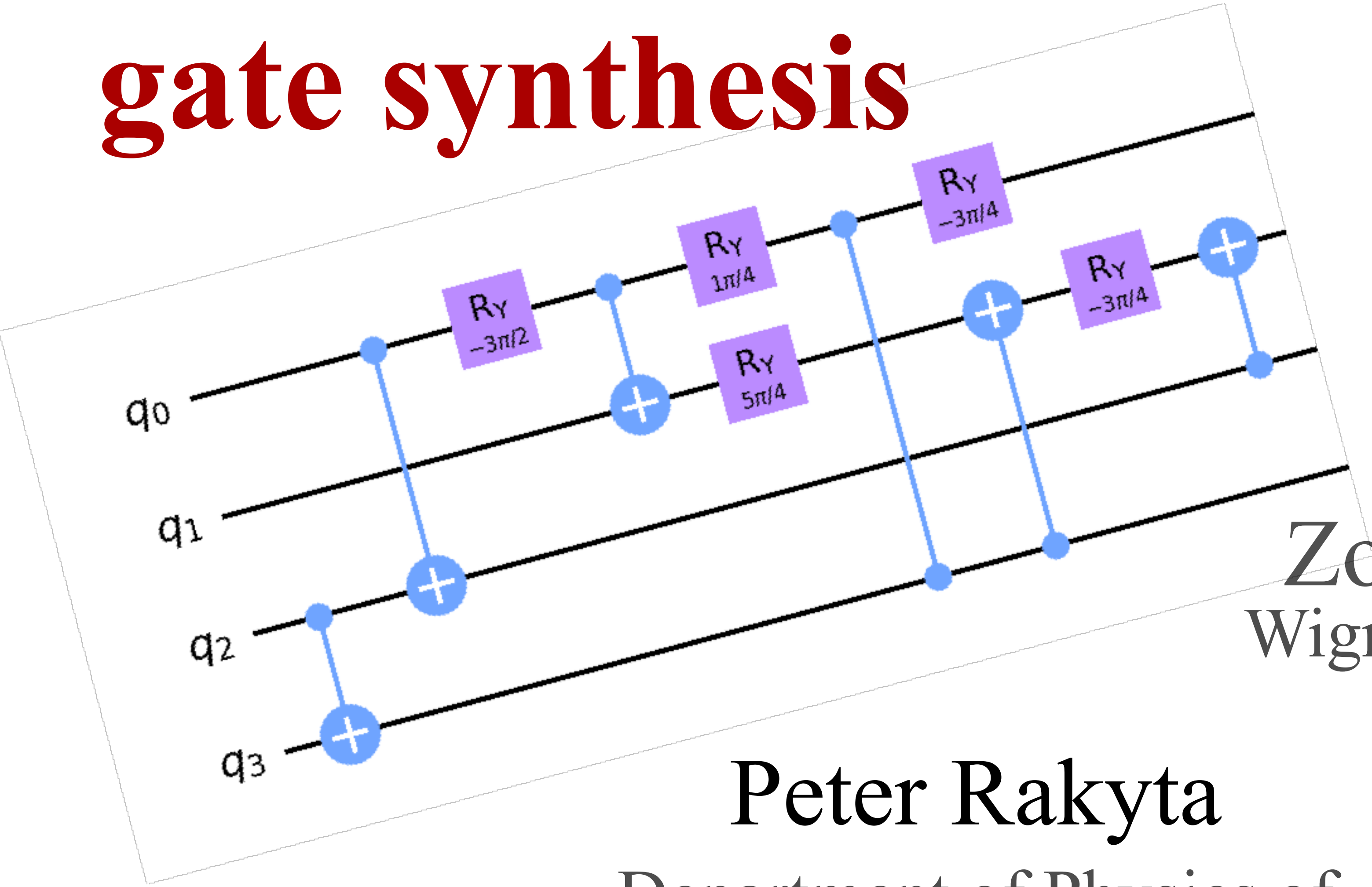
Quantum Information
National Laboratory
HUNGARY

Approximate quantum gate synthesis



Zoltán Zimborás
Wigner Research Center
for Physics

Peter Rakyta
Department of Physics of
Complex Systems

ELTE
EÖTVÖS LORÁND
UNIVERSITY

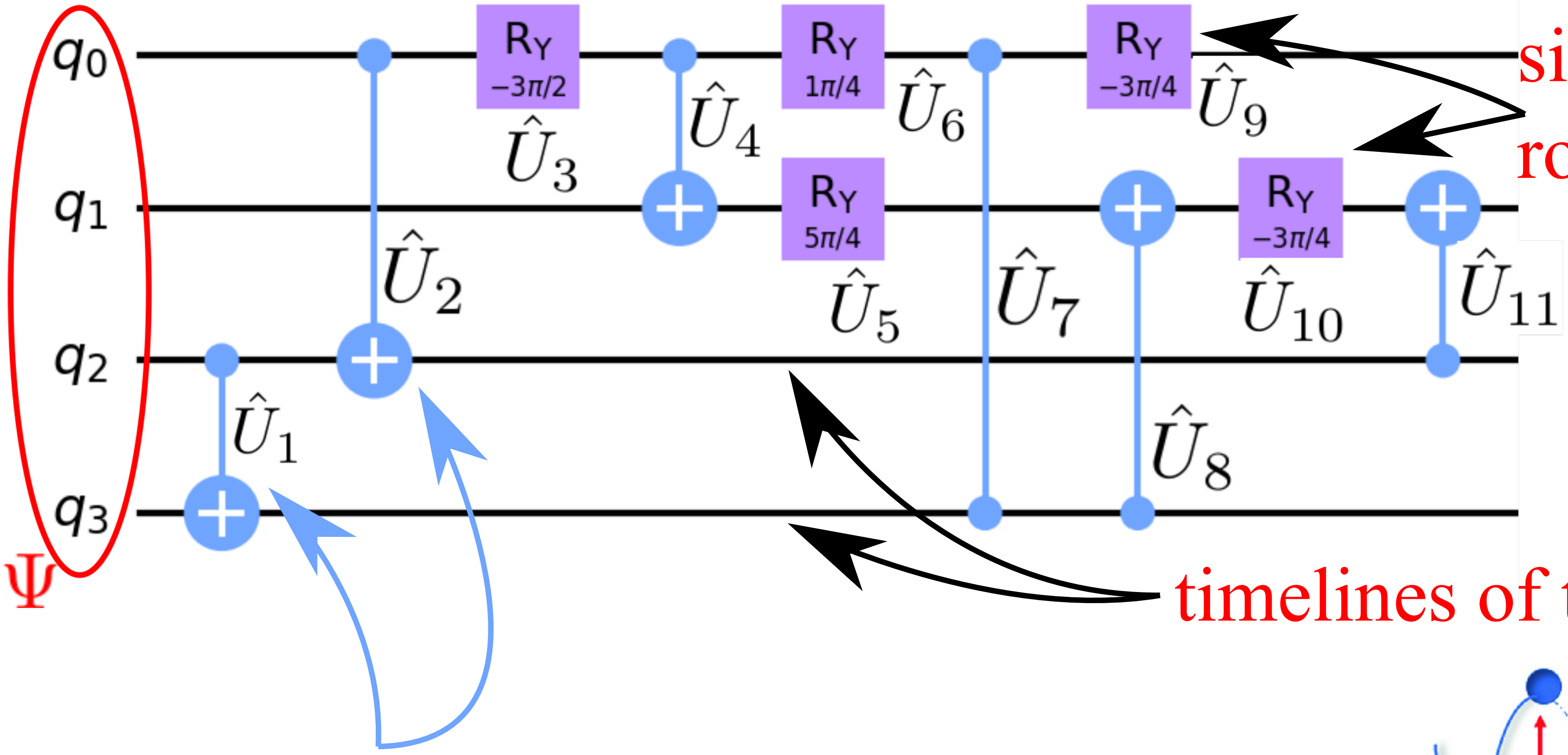
WIGNER

Quantum gate decomposition

Quantum Information
National Laboratory
HUNGARY

quantum program (unitary)

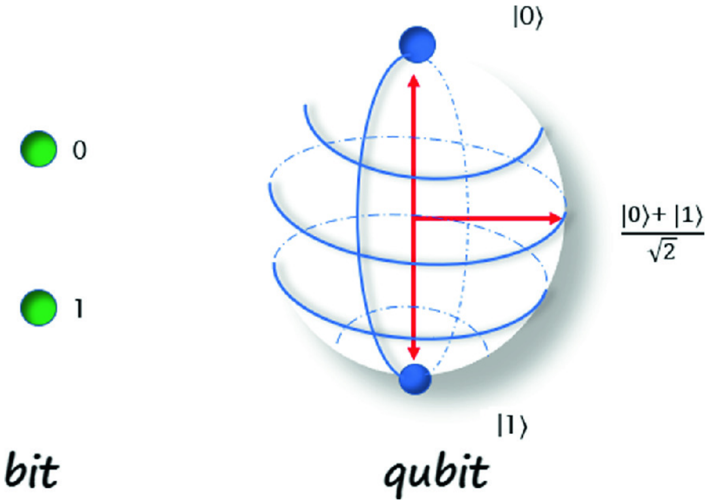
$$\hat{U} = \hat{U}_{11} \cdot \hat{U}_{10} \cdot \hat{U}_9 \cdot \hat{U}_8 \cdot \hat{U}_7 \cdot \hat{U}_6 \cdot \hat{U}_5 \cdot \hat{U}_4 \cdot \hat{U}_3 \cdot \hat{U}_2 \cdot \hat{U}_1$$



single qubit rotations

timelines of the qubits

controlled not gates



ELTE
EÖTVÖS LORÁND
UNIVERSITY

Wigner

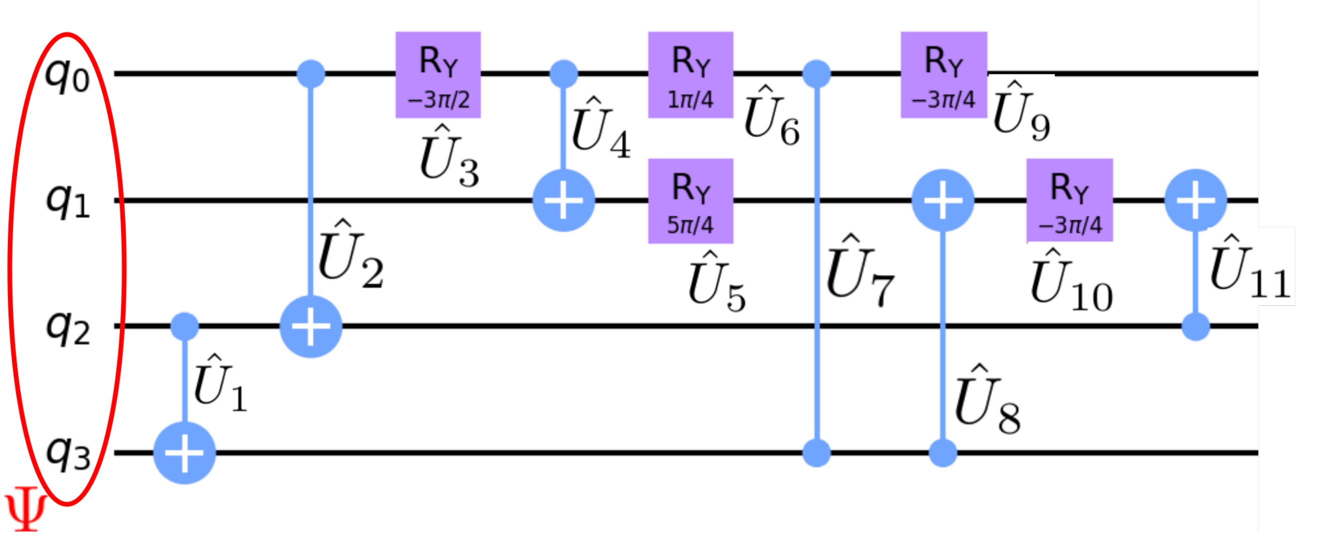
Quantum gate decomposition

The gate decomposition is a two-fold problem of

- Combinatorial problem of placing the gates
- Optimization problem of the continuous parameters

How to measure the distance from the quantum program U?

$$\hat{U} = \hat{U}_{11} \cdot \hat{U}_{10} \cdot \hat{U}_9 \cdot \hat{U}_8 \cdot \hat{U}_7 \cdot \hat{U}_6 \cdot \hat{U}_5 \cdot \hat{U}_4 \cdot \hat{U}_3 \cdot \hat{U}_2 \cdot \hat{U}_1$$



ELTE
EÖTVÖS LORÁND
UNIVERSITY

How close is an approximation to the exact one?

exact evolution: U

$$|\psi(U)\rangle := U|\psi\rangle$$

approximate evolution: V

$$|\psi(V)\rangle := V|\psi\rangle$$

The fidelity of the approximation:

$$\overline{F}(U,V) := \int_{\psi} |\langle\psi(V)|\psi(U)\rangle|^2 \, \mathrm{d}\psi$$

average taken over the Haar distribution

The cost function of the optimization:

$$C_{HST}(U,V) = 1 - \frac{1}{d^2}\left|\mathrm{Tr}\left(V^{\dagger}U\right)\right|^2$$

Hilbert-Schmidt test

dimension of the Hilbert-space

$$\overline{F}(U,V) = 1 - \frac{d}{d+1} C_{HST}(U,V)$$

EÖTVÖS LORÁND UNIVERSITY

exact decmposition: $C_{HST}(U,V) = 0 \quad \overline{F}(U,V) = 1$

How close is an approximation to the exact one?

Frobenius-norm based fidelity

$$\|A\|_{\mathrm{F}} = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2 \right)^{\frac{1}{2}}$$

The cost function of the optimization:

$$f(U,V) = \frac{1}{2} \|V - U\|_F^2 = d - \mathrm{Re}\left[\mathrm{Tr}(U^\dagger V)\right]$$

The Fidelity:

$$\overline{F}_F(U,V) = 1 - \frac{d}{d+1} + \frac{1}{d(d+1)} \left(d - f(U,V)\right)^2$$

$$\overline{F}_F(U,V) \le \overline{F}(U,V)$$

"Best Approximate Quantum Compiling Problems"

Liam Madden (University of Colorado),
Andrea Simonetto (IBM Research Ireland)
arXiv:2106.05649

Optimized quantum circuit synthesis

Quantum Information
National Laboratory
HUNGARY

How to find a more optimal gate decomposition?

- fewest gate count?
- smallest depth?

Available gate decomposition utilities:

- Quantum Fast Approximate Synthesis Tool (QFAST)
- QSearch + LEAP

(Lawrence Berkeley National Laboratory)

BERKELEY LAB

- UniversalQCompiler (incorporated into QISKIT)

(ETH Zürich, University of York, TUM)

T|ket>: A Retargetable Compiler for NISQ Devices

(Cambridge Quantum Computing Ltd., University of Strathclyde

ELTE
EÖTVÖS LORÁND
UNIVERSITY

WIGNER

Heuristics for Quantum Compiling with a Continuous Gate Set (QSearch)

Quantum Information
National Laboratory
HUNGARY

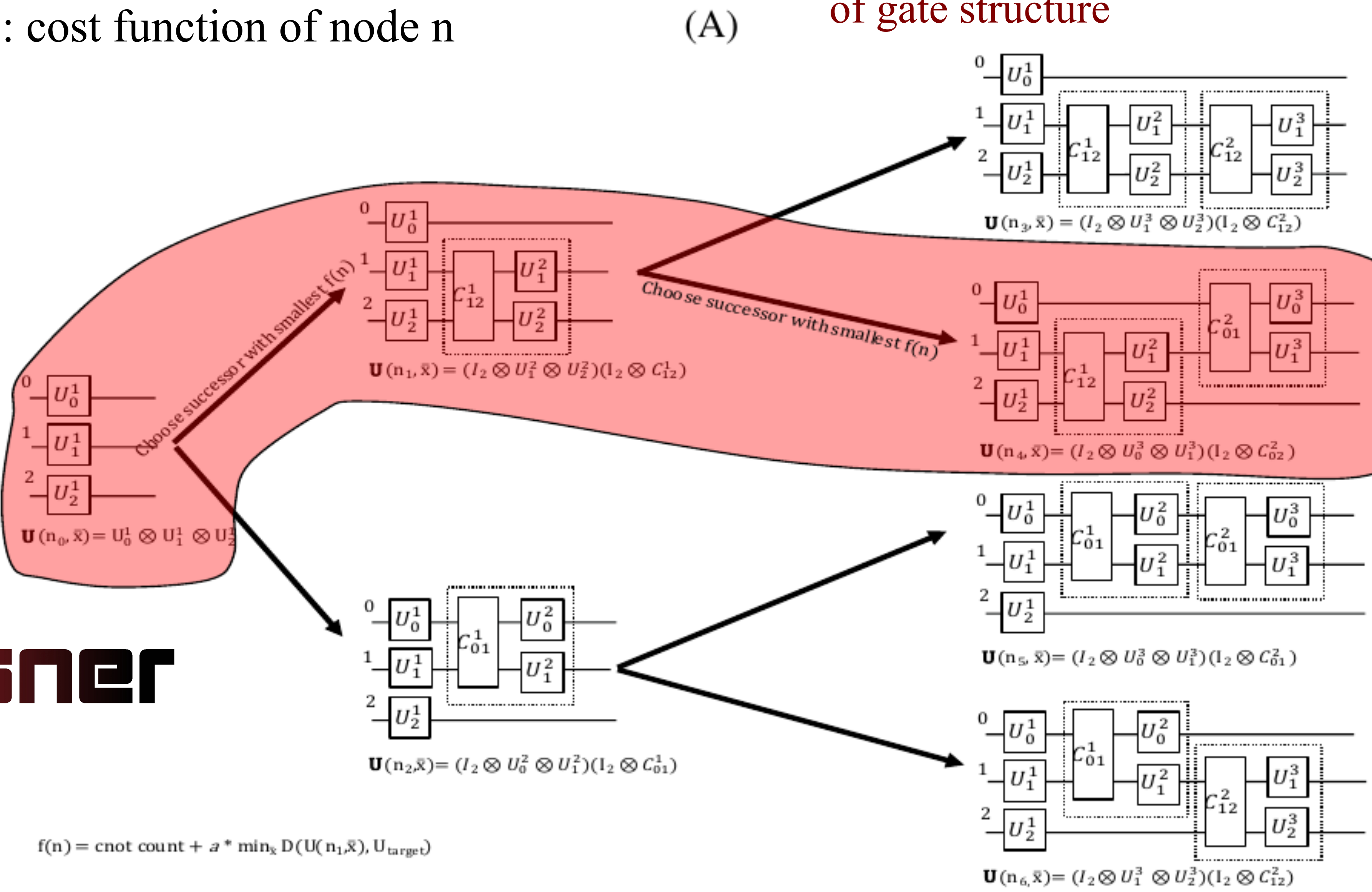
a) Optimization of continuous parameters in U3 gates

b) **Combinatorial optimization problem** of gate structure

A* Search Strategy

partial solutions in graph nodes

f(n): cost function of node n



$f(n) = \text{cnot count} + a * \min_{\bar{x}} D(U(n_1, \bar{x}), U_{\text{target}})$

Quantum Fast Approximate Synthesis Tool (QFAST)

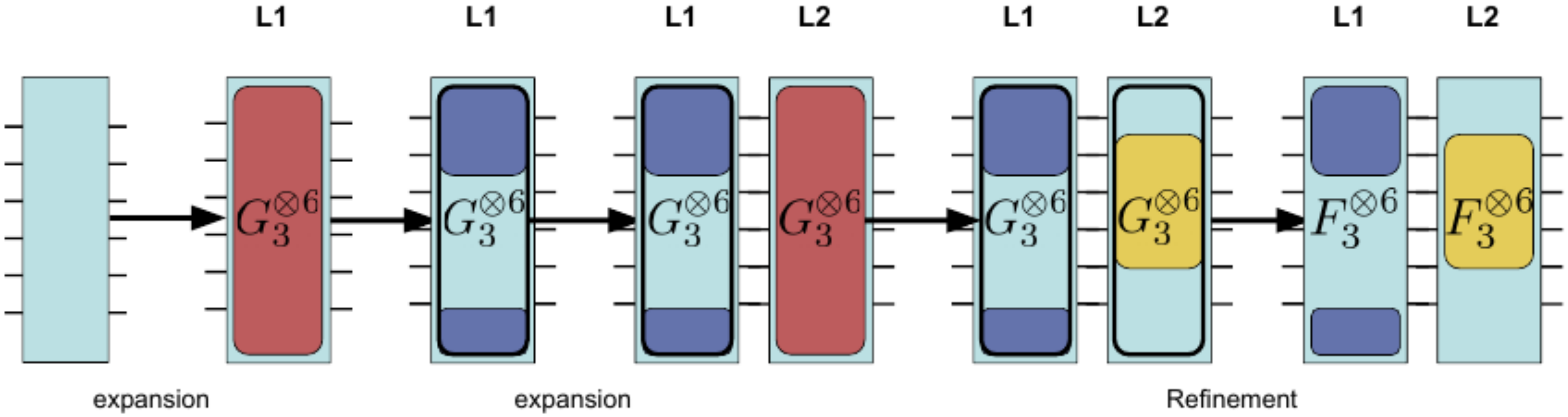
Quantum Information
National Laboratory
HUNGARY

n-qubit unitary $\longrightarrow \frac{n}{2}$ -qubit unitaries $\longrightarrow \frac{n}{4}$ -qubit unitaries

$\longrightarrow$ 1 and 2-qubit unitaries

$$U(2^n) = \{e^{\mathrm{i}(\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}^{\otimes n})} \mid \boldsymbol{\alpha} \in \mathbb{R}^{4^n}\}$$

$$\boldsymbol{\sigma}^{\otimes n} = \{\sigma_j \otimes \sigma_k \mid \sigma_j \in \boldsymbol{\sigma}, \sigma_k \in \boldsymbol{\sigma}^{\otimes n-1}\} \qquad \boldsymbol{\sigma} = \{\sigma_i, \sigma_x, \sigma_y, \sigma_z\}$$



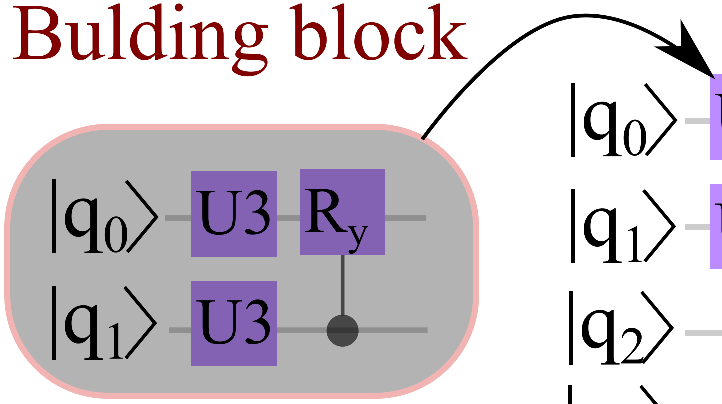
ELTE
EÖTVÖS LORÁND
UNIVERSITY

WIGNER

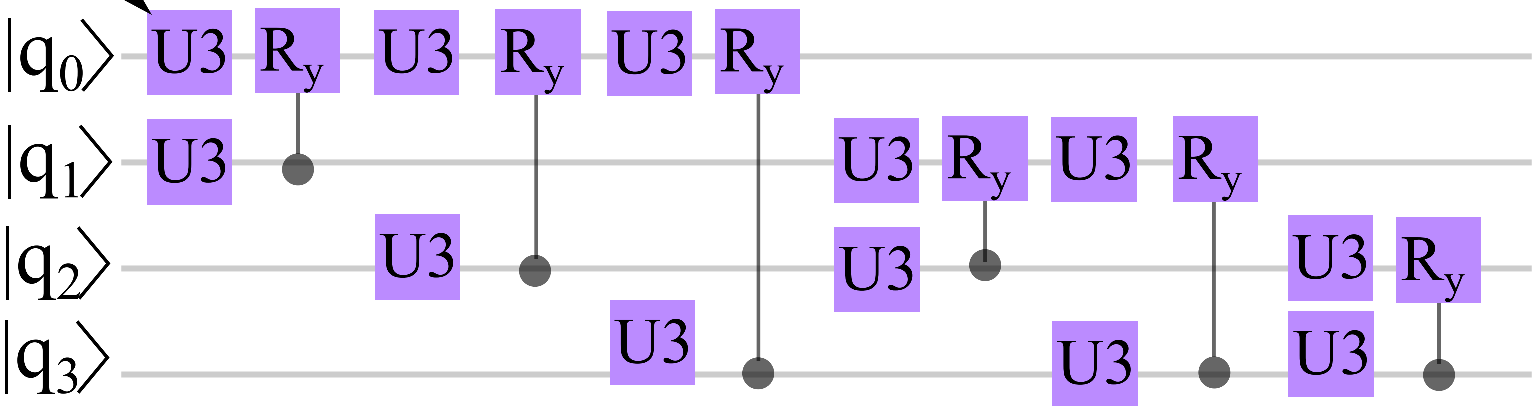
Adaptive quantum gate decomposition (SQUANDER)

Quantum Information
National Laboratory
HUNGARY

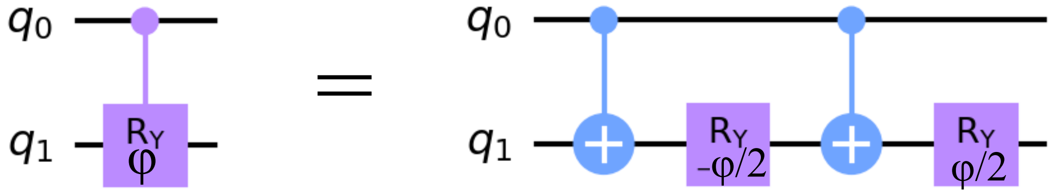
Bulding block



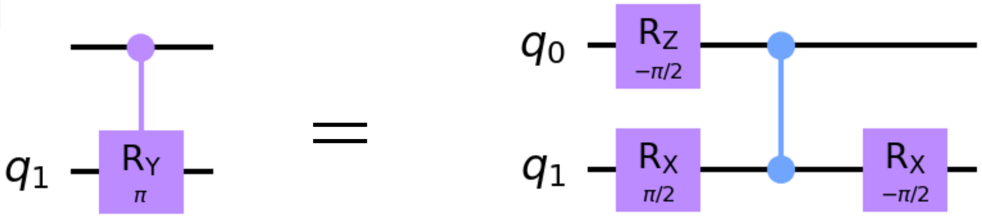
Decomposing gate structure

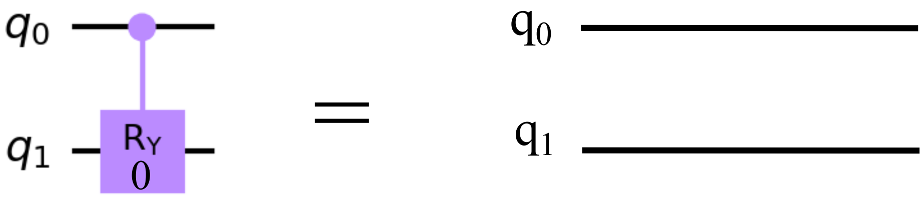


Expansion of controlled R_y rotations



Special case





ELTE
EÖTVÖS LORÁND
UNIVERSITY

wigner

SQUANDER vs QFAST vs QSearch

a) In the benchmark we tested the decomposition of 3, 4 and 5-qubit unitaries from online database containing series of circuits published as part of the Qiskit Developer Challenge, a public competition to design a better routing algorithm.

b) quantum circuits of well known algorithms:
Grover search,
Quantum Fourier Transformation (QFT)
Quantum Approximate Optimization Algorithm (QAOA),
Quantum variational eigensolver (VQE)

Gate synthesis benchmark

Quantum Information
National Laboratory
HUNGARY

File name	Initial	QISKIT	SQUANDER		QFAST		QSEARCH	
	$CNOT$	$CNOT$	$CNOT$	$\overline{T}\,[s]$	$CNOT$	$\overline{T}\,[s]$	$CNOT$	$\overline{T}\,[s]$
4gt5_77	58	**338**	**23**	1293	**26**	332	-	-
4gt13_91	49	**187**	**23**	1296	**25**	732	**48**	2324
ham3_102	11	**15**	**6**	4.9	**7**	3.2	**8**	2.6
4gt5_76	46	**529**	**24**	1711	**29**	476	-	-
alu-v0_26	38	**204**	**23**	7900	**42**	912	**29**	9284
miller_11	23	**18**	**8**	7	**9**	5.4	**10**	4.5
rd32_v1_68	16	**66**	**9**	23.9	**13**	21.6	**13**	615
4mod5-v0_20	10	**526**	**9**	3650	**17**	166	**16**	14508
alu-v0_27	17	**212**	**17**	3452	**30**	674	**34**	3801
mod5mils_65	16	**73**	**12**	11162	**20**	405	-	-
ex-1_166	9	**20**	9	4.4	8	4.7	8	5.9
decod24-v1_41	38	**130**	**20**	2414	**36**	413	**24**	349
alu-v3_34	24	**237**	**25**	6090	**37**	1814	**27**	7834
3_17_13	17	**23**	**7**	6.5	**9**	4.2	**9**	4.3
4gt11_84	9	**163**	**9**	642	**20**	318	-	-
decod24-v0_38	23	**48**	**14**	62	**23**	58	**15**	285
4mod5-v0_19	16	**75**	**13**	701	**21**	375	-	-
4mod5-v1_22	11	**168**	**9**	962	**13**	52	**17**	82

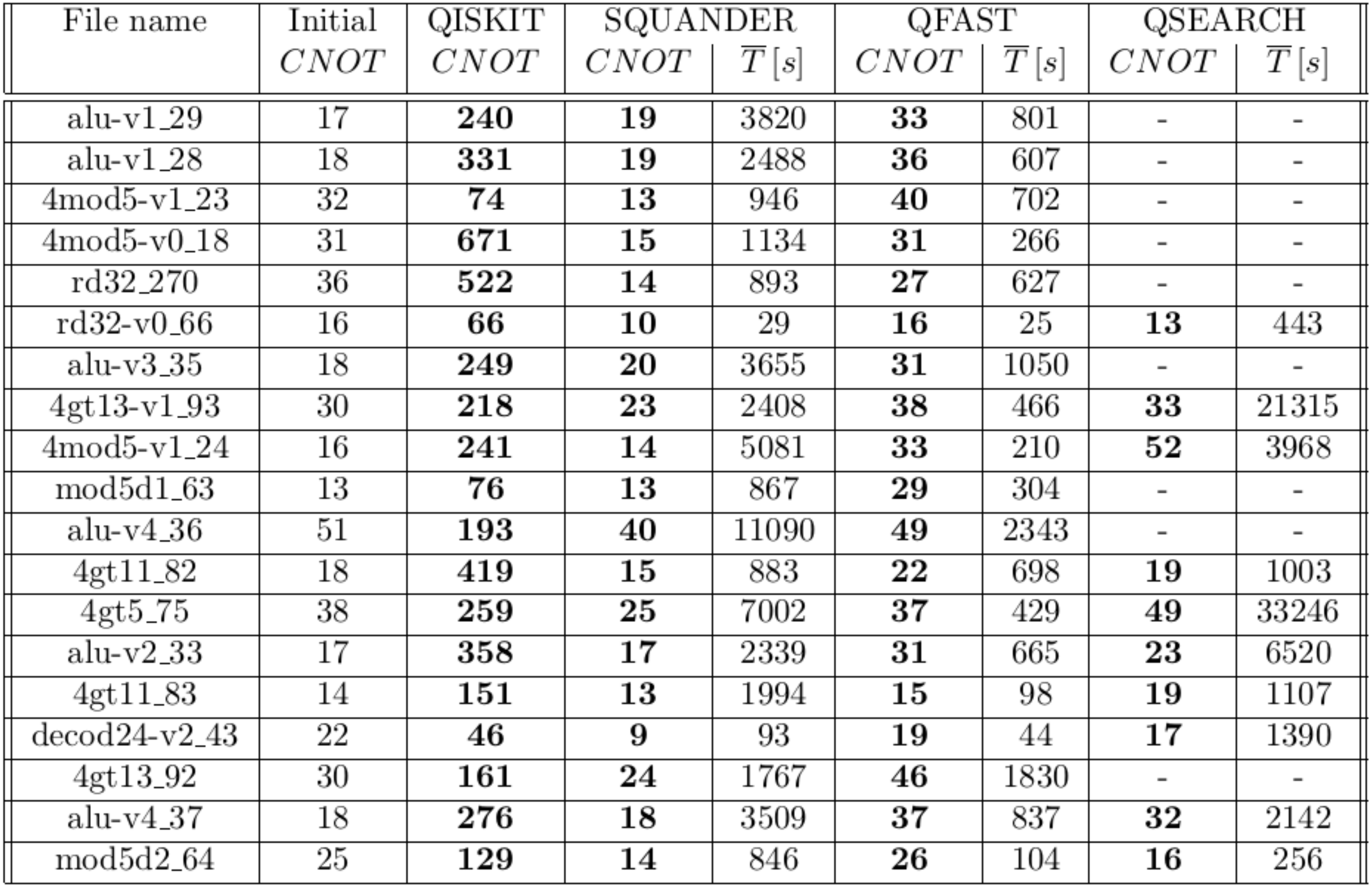
EÖTVÖS LORÁND
UNIVERSITY

optimization error: $f < 10^{-8}$

WIGNER

Gate synthesis benchmark

Quantum Information
National Laboratory
HUNGARY



File name	Initial	QISKIT	SQUANDER		QFAST		QSEARCH	
	$CNOT$	$CNOT$	$CNOT$	$\overline{T}\,[s]$	$CNOT$	$\overline{T}\,[s]$	$CNOT$	$\overline{T}\,[s]$
alu-v1_29	17	**240**	**19**	3820	**33**	801	-	-
alu-v1_28	18	**331**	**19**	2488	**36**	607	-	-
4mod5-v1_23	32	**74**	**13**	946	**40**	702	-	-
4mod5-v0_18	31	**671**	**15**	1134	**31**	266	-	-
rd32_270	36	**522**	**14**	893	**27**	627	-	-
rd32-v0_66	16	**66**	**10**	29	**16**	25	**13**	443
alu-v3_35	18	**249**	**20**	3655	**31**	1050	-	-
4gt13-v1_93	30	**218**	**23**	2408	**38**	466	**33**	21315
4mod5-v1_24	16	**241**	**14**	5081	**33**	210	**52**	3968
mod5d1_63	13	**76**	**13**	867	**29**	304	-	-
alu-v4_36	51	**193**	**40**	11090	**49**	2343	-	-
4gt11_82	18	**419**	**15**	883	**22**	698	**19**	1003
4gt5_75	38	**259**	**25**	7002	**37**	429	**49**	33246
alu-v2_33	17	**358**	**17**	2339	**31**	665	**23**	6520
4gt11_83	14	**151**	**13**	1994	**15**	98	**19**	1107
decod24-v2_43	22	**46**	**9**	93	**19**	44	**17**	1390
4gt13_92	30	**161**	**24**	1767	**46**	1830	-	-
alu-v4_37	18	**276**	**18**	3509	**37**	837	**32**	2142
mod5d2_64	25	**129**	**14**	846	**26**	104	**16**	256

UNIVERSITY

optimization error: $f < 10^{-8}$

SQUANDER benchmark

In the 21% percent of the experiments SQUANDER decreased the number of the CNOT gates by more than 50%

In 68% of the examples the compression was more than 10%.

In exchange of larger execution time...

Conclusions and outlook

We have designed an adaptive circuit compression algorithm providing the fewest gatecount in the decomposition.

Aiming to reduce the execution time by:

- Using first order optimization method
  (currently second-order BBFG method is used)
- Make faster the evaluation of the cost function using data-flow engines.

Aknowledgement

This research was supported by the Ministry of Innovation and Technology and the National Research, Development and Innovation Office within the Quantum Information National Laboratory of Hungary

We acknowledge the computational resources provided by the Wigner Scientific Computational Laboratory (WSCLAB)

(the formerWigner GPU Laboratory)

contact: Peter Rakyta, peter.rakyta@ttk.elte.hu