Department of
Hydrodynamic
Systems

# GPU accelerated investigation of a dual-frequency driven nonlinear oscillator


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# International Co-operation

- Prof. Werner Lauterborn
  - Independent WoS citation: 5923
  - Drittes Physikalisches Institut (DPI), Georg-August-Universität Göttingen, Germany

- Dr. Robert Mettin
  - Independent WoS citation: 1185
  - Drittes Physikalisches Institut (DPI), Georg-August-Universität Göttingen

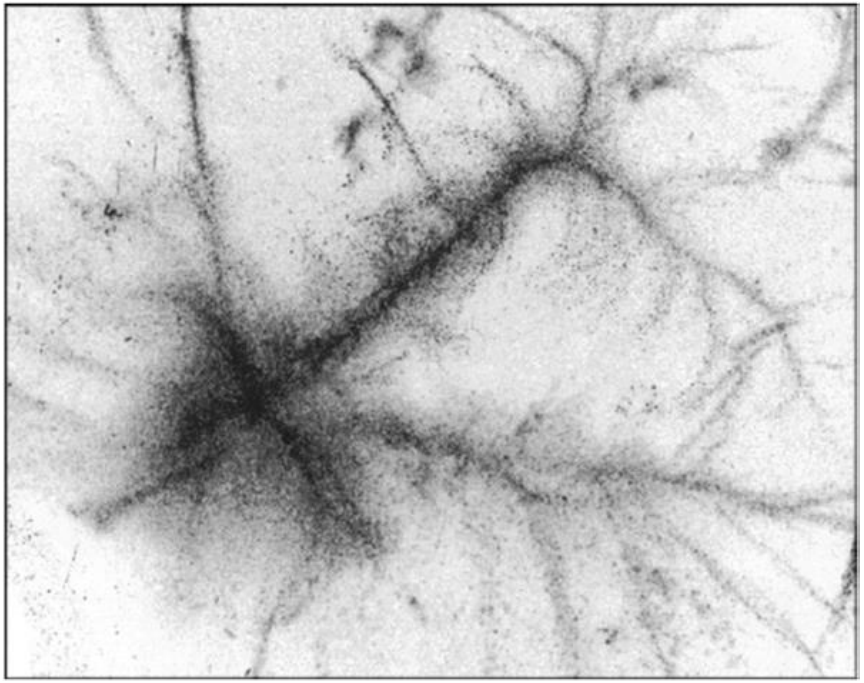
- Prof. Ulrich Parlitz
  - Independent WoS citation: 6307
  - Biomedical Physics Group, Max Planck Institute for Dynamics and Self-Organization (MPI), and Institute for Nonlinear Dynamics, Georg-August-Universität

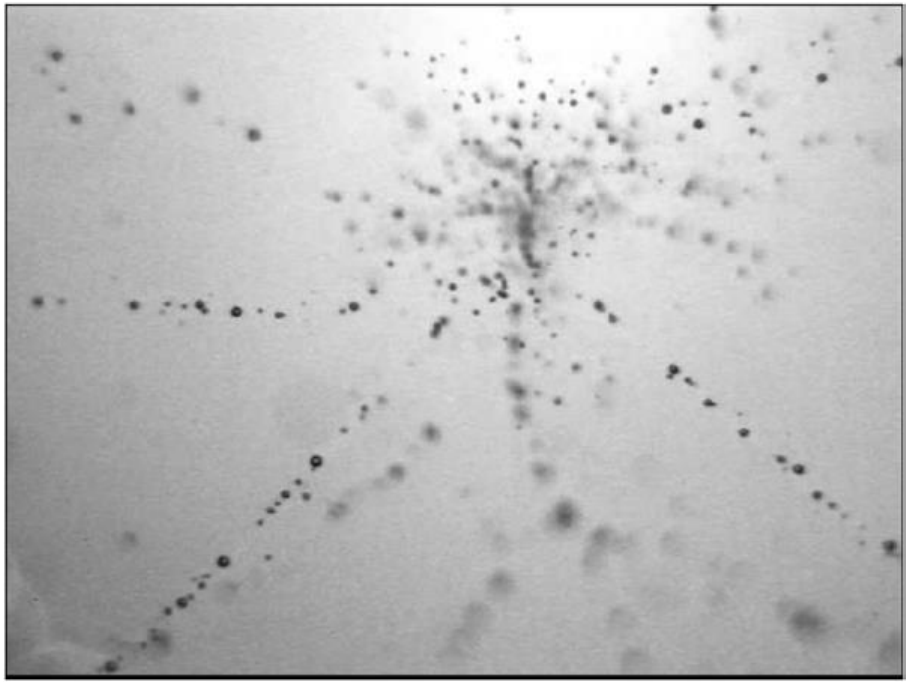
# Acoustic Cavitation

- *Bubble clusters in moving and standing sound waves:*

# Acoustic Cavitation

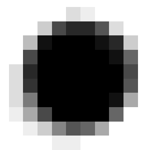
➢ *Magnified bubble clusters:*





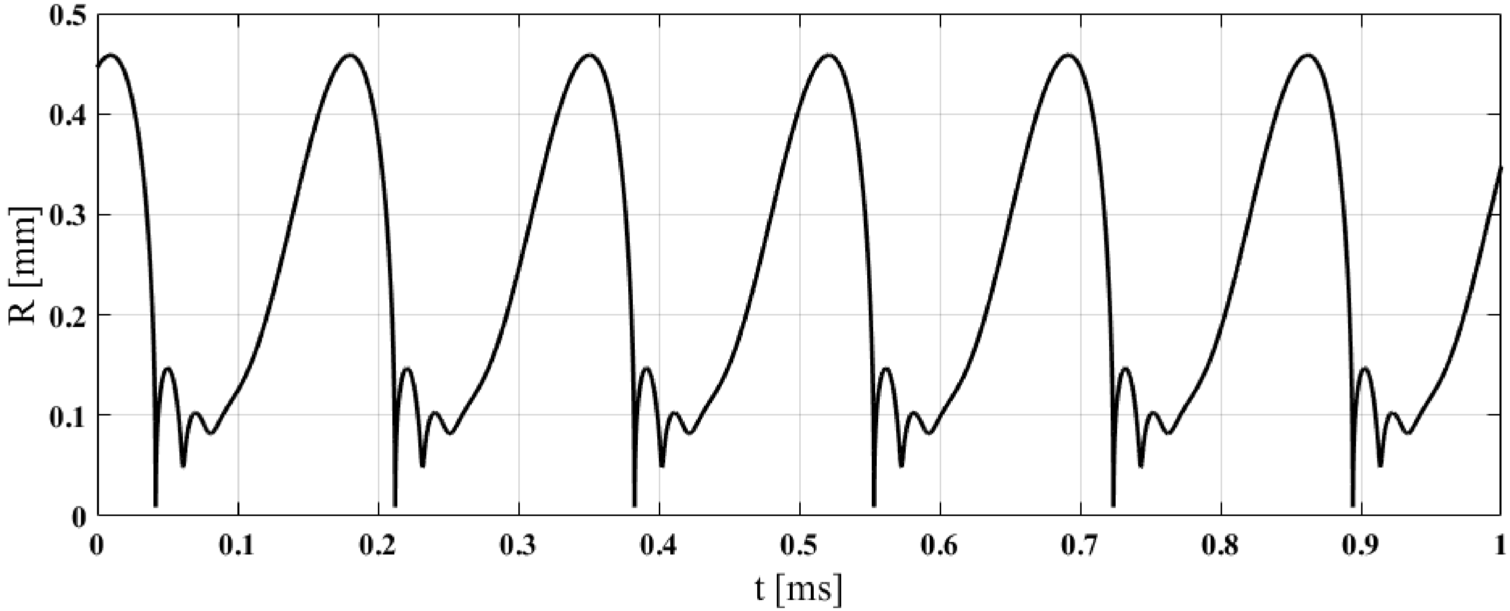
# Acoustic Cavitation

- *Single bubble in a standing sound field:*
  - *Frequency: 25,1 kHz*
  - *Pressure amplitude: approx. 2 bar*
  - *Frame rate: 775 kfps*



# Acoustic Cavitation

➤ *Tipical bubble radius vs. time curve:*




**wall velocity**
$\dot{R}_{max} \approx 10^3$ m/s

→

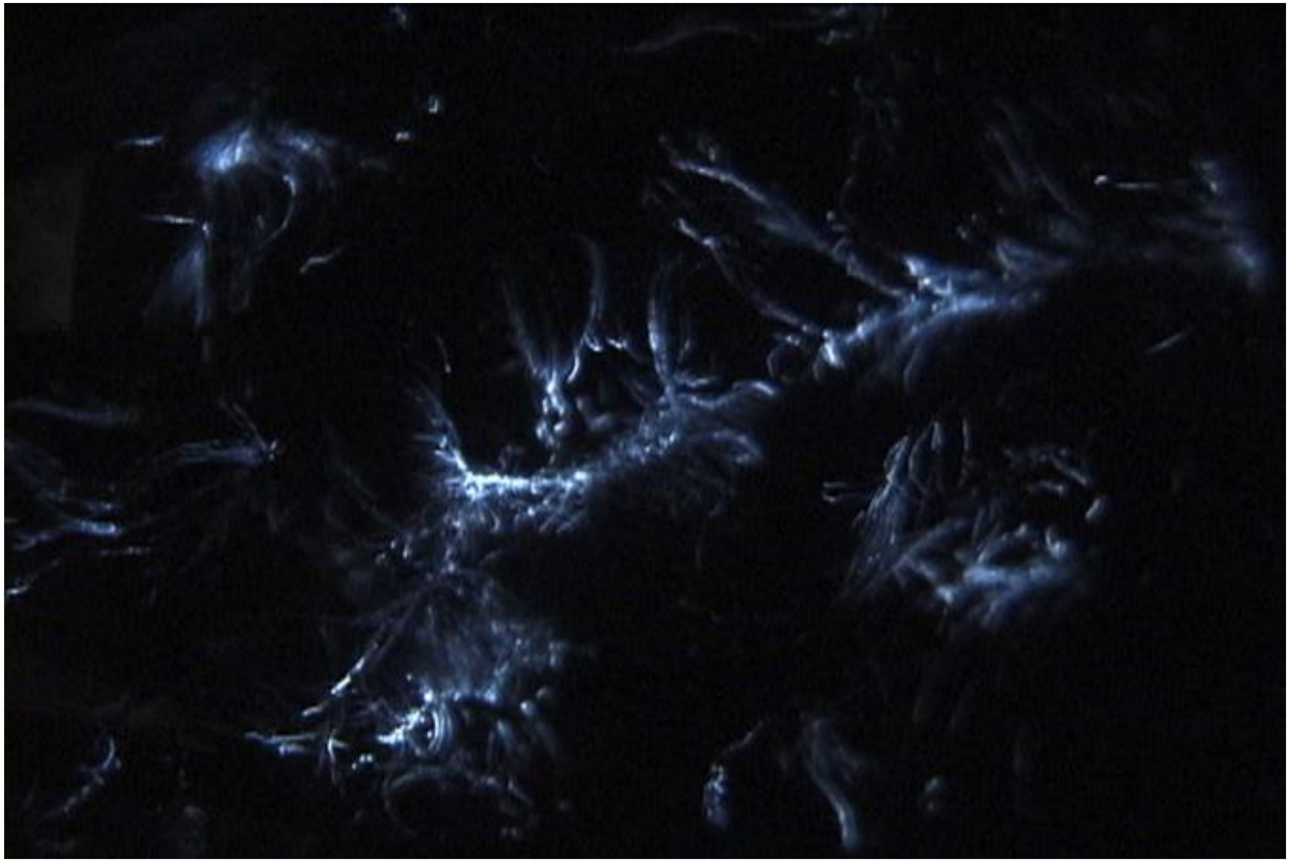
**extreme conditions**
$T_{max} \approx 10^4$ K

→

**shock waves**

# Acoustic Cavitation

- *Sonoluminescence:*



# Applications

- *Applications in ultrasonic technology:*

**Reducing the chain length of polymers and creation of new copolymers**
(Konaganti and Madaras, 2010)

**Increasing the efficiency of heterogeneous catalysis**
(Disselkamp et al., 2004)

**Szonochemistry**
(Suslick, K. S. 1990)

- *Speciality: application of dual-frequency*

# The Bubble Model

- *Keller—Miksis equation :*

$$\left(1-\frac{\dot{R}}{c_L}\right)R\ddot{R}+\left(1-\frac{\dot{R}}{3c_L}\right)\frac{3}{2}\dot{R}^2=\left(1+\frac{\dot{R}}{c_L}+\frac{R}{\rho_L c_L}\frac{d}{dt}\right)\frac{(p_L-p_\infty)}{\rho_L}$$

- *Dual-frequency excitation:*

$$p_\infty(t)=P_\infty+p_{A1}\sin(\omega_1 t)+p_{A2}\sin(\omega_2 t+\Theta)$$

- *Relative frequencies:*

$$\omega_{R1}\ \frac{\omega_1}{\omega_0}$$

$$\omega_{R2}\ \frac{\omega_2}{\omega_0}$$

# The Bubble Model

- *Keller—Miksis equation:*

$$\dot{y}_1 = y_2 = F_1(y_1, y_2, \tau)$$

$$\dot{y}_2 = \frac{N}{D} = F_2(y_1, y_2, \tau)$$

- *Numerator and the Denominator:*

$$N = (C_0 + C_1 y_2)\left(\frac{1}{y_1}\right)^{C_{10}} - C_2(1 + C_9 y_2) - C_3 \frac{1}{y_1} - C_4 \frac{y_2}{y_1} - \left(1 - C_9 \frac{y_2}{3}\right)\frac{3y_2^2}{2}$$
$$- (C_5 \sin(2\pi\tau) + C_6 \sin(2\pi C_{11}\tau + C_{12}))(1 + C_9 y_2)$$
$$- y_1(C_7 \cos(2\pi\tau) + C_7 \cos(2\pi C_{11}\tau + C_{12}))$$
$$D = y_1 - C_9 y_1 y_2 + C_4 C_9$$

- *13 constants $C_o$ to $C_{12}$, frequency ratio and phase angle:*

$$C_{11} = \frac{\omega_2}{\omega_1}, \quad C_{12} = \Theta$$

# The Bubble Model

- *Linearized system in polar coordinates:*

$$\dot{y}_3 = y_3\left(\left(1+g_1\right)\sin y_4 \cos y_4 + g_2 \sin^2 y_4\right)$$
$$\dot{y}_4 = -\sin^2 y_4 + \left(g_1 \cos y_4 + g_2 \sin y_4\right)\cos y_4$$

- *where:*

$$g_1 = \frac{\partial F_2}{\partial y_1} = \frac{\frac{\partial N}{\partial y_1} D - N \frac{\partial D}{\partial y_1}}{D^2}$$

$$g_2 = \frac{\partial F_2}{\partial y_2} = \frac{\frac{\partial N}{\partial y_2} D - N \frac{\partial D}{\partial y_2}}{D^2}$$

- *"Expensive" operations:*

$$\frac{1}{y_1}, \quad y_1^{C_{10}}, \quad \sin\cos\left(2\pi\tau\right), \quad \sin\cos\left(2\pi C_{11}\tau\right), \quad \sin\cos y_4$$

# Solution Technique

- *Investigated parameter space:*

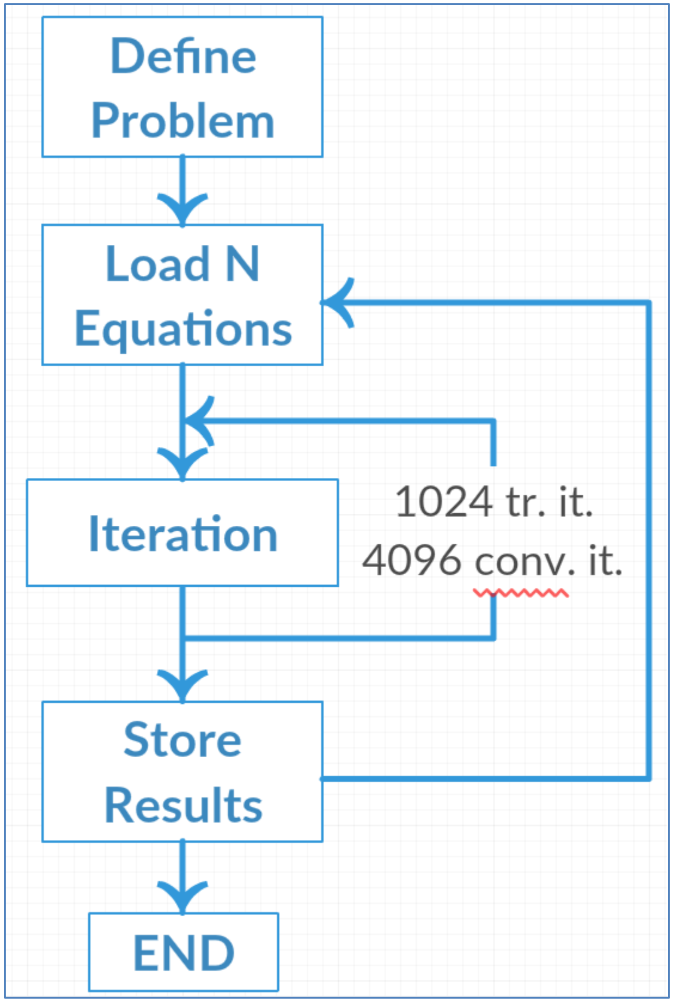
  $P_{A1},\quad P_{A2},\quad \omega_{\mathrm{R1}},\quad \omega_{\mathrm{R2}},\quad (\Theta = 0)$

- *Platform:*
  - *GPGPU (General Purpose Graphics Processing Units)*
    - *Titan Black (Kepler, 1707 GFLOPS, number: 1)*
    - *Tesla K20 (Kepler, 1175 GFLOPS, number: 2)*
    - *Tesla M2050 (Fermi, 515 GFLOPS, number: 8)*
    - *Tesla P100 (Pascal, 4670 GFLOPS, ???)*

- *Numerical algorithm:*
  - *Runge-Kutta-Cash-Karp (explicit, adaptive)*

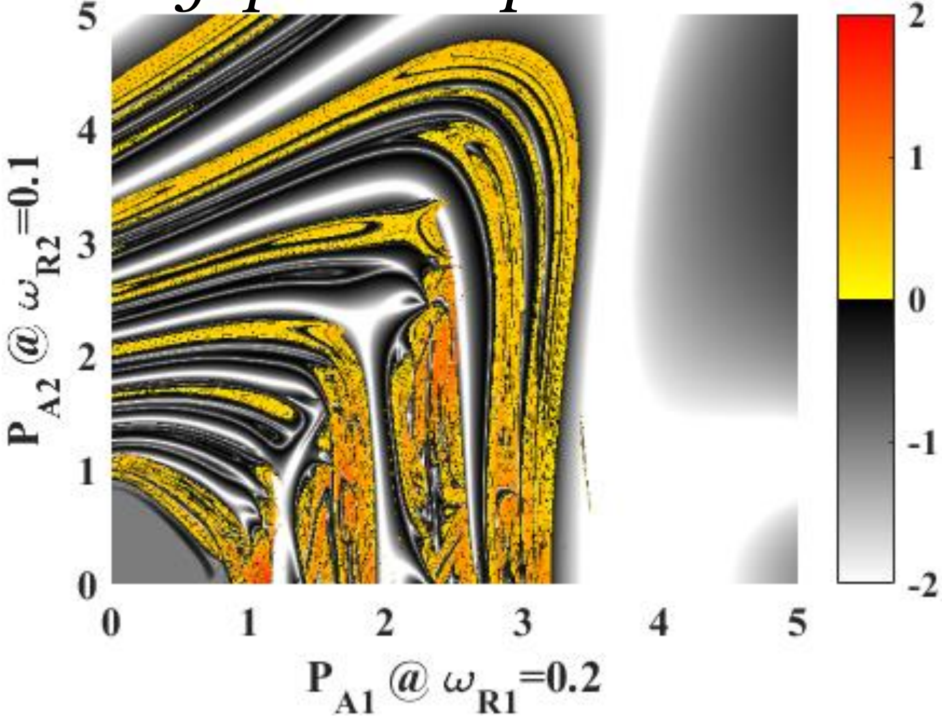
# Implementation

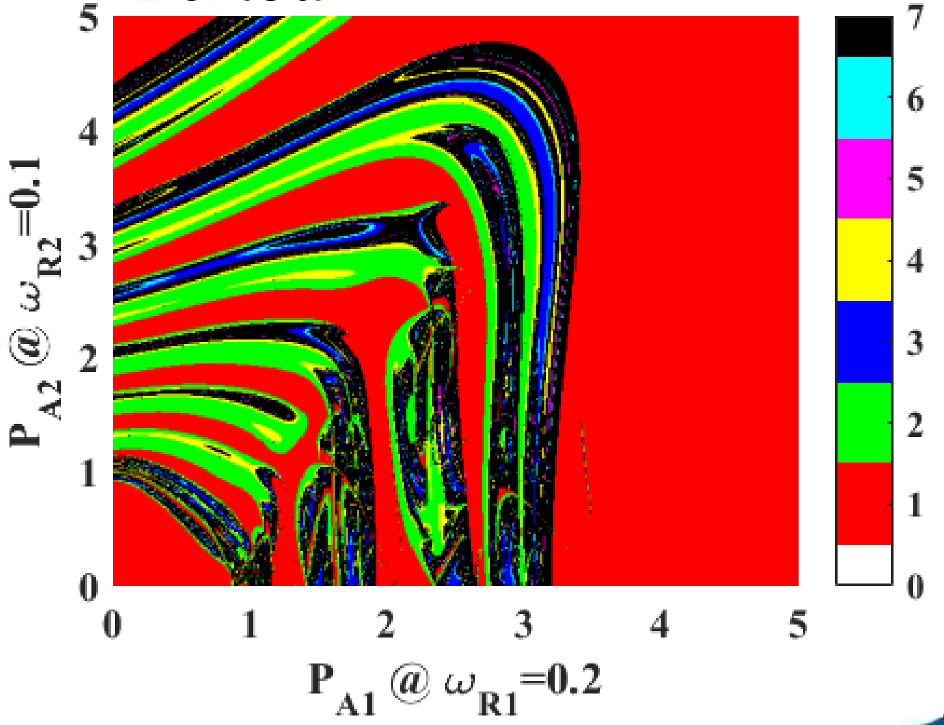
- *1 thread – 1 system*
- *RK constants: const. mem.*
- *Shared mem.: not used*
- *Regs./thr.: 128/63*
- *Thr./blocks: 64*
- *Occupacy: 25%*
- *Arthm.: >95%*
- *Adaptive: warp div.*



# High Resolution Plots

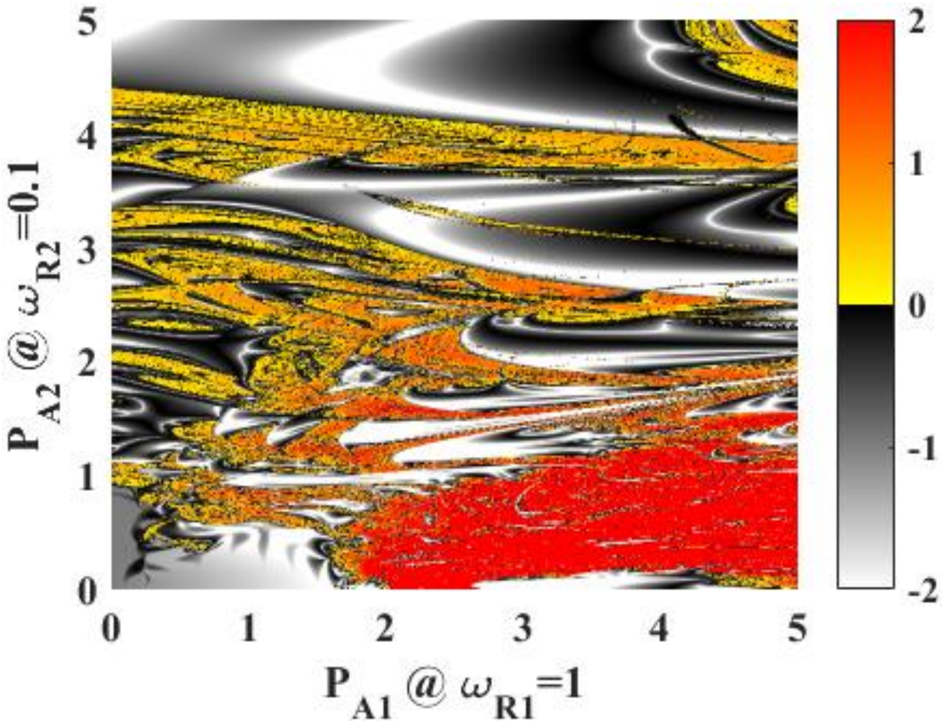
- *Control parameters:* $P_{A1}$, $P_{A2}$
  - *Range: 0-5 bar, Resolution: 501*
- *Relative frequencies:* $\omega_{R1,2} = 1/10,\ 1/5,\ 1/3,\ 1/2,\ 1,\ 2,\ 3,\ 5,\ 10$
- *10 randomized initial conditions*
  - *Lyapunov exponent*
  - *Period*



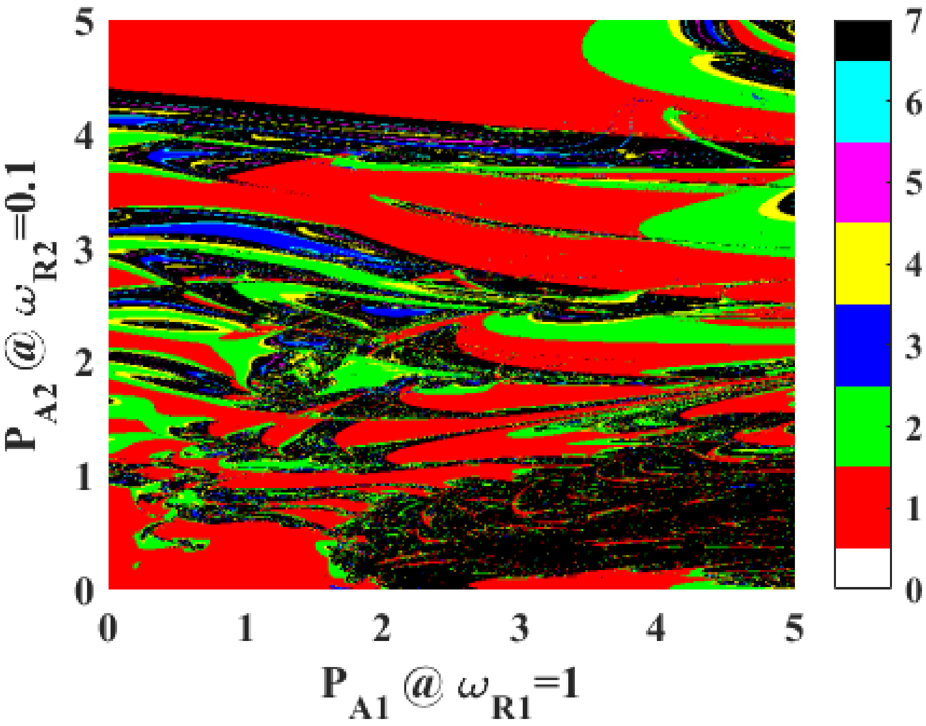



# High Resolution Plots

- *Lyapunov exponent*




- *Period*



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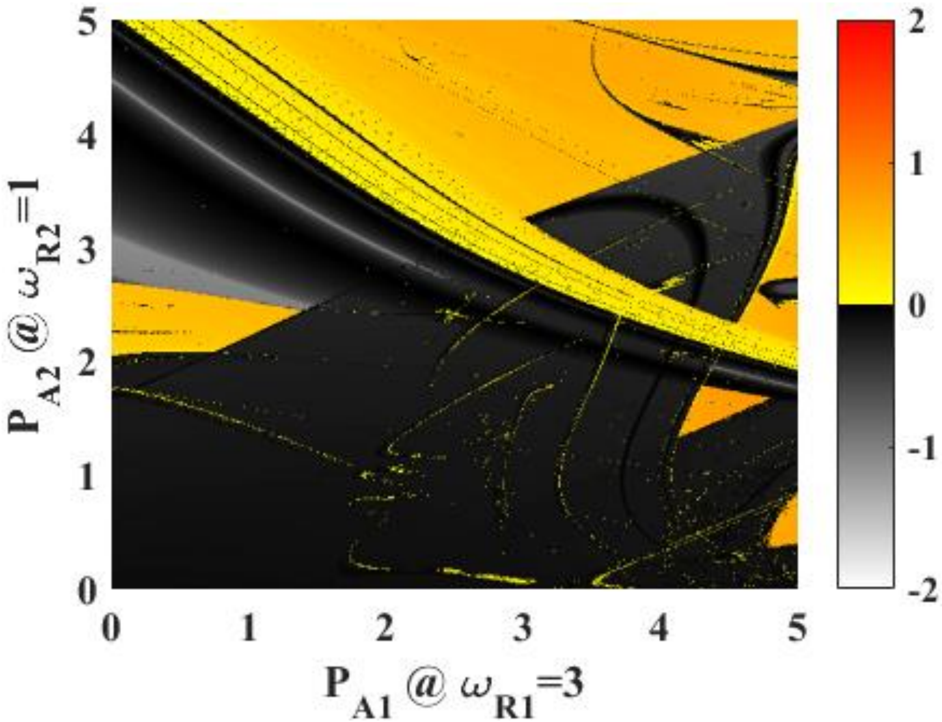
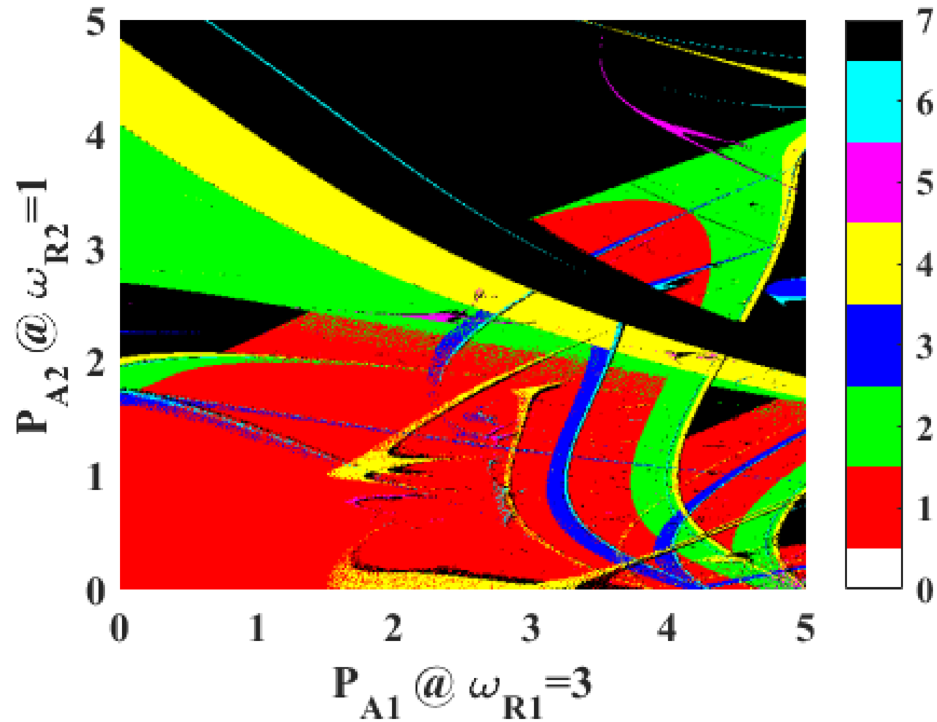
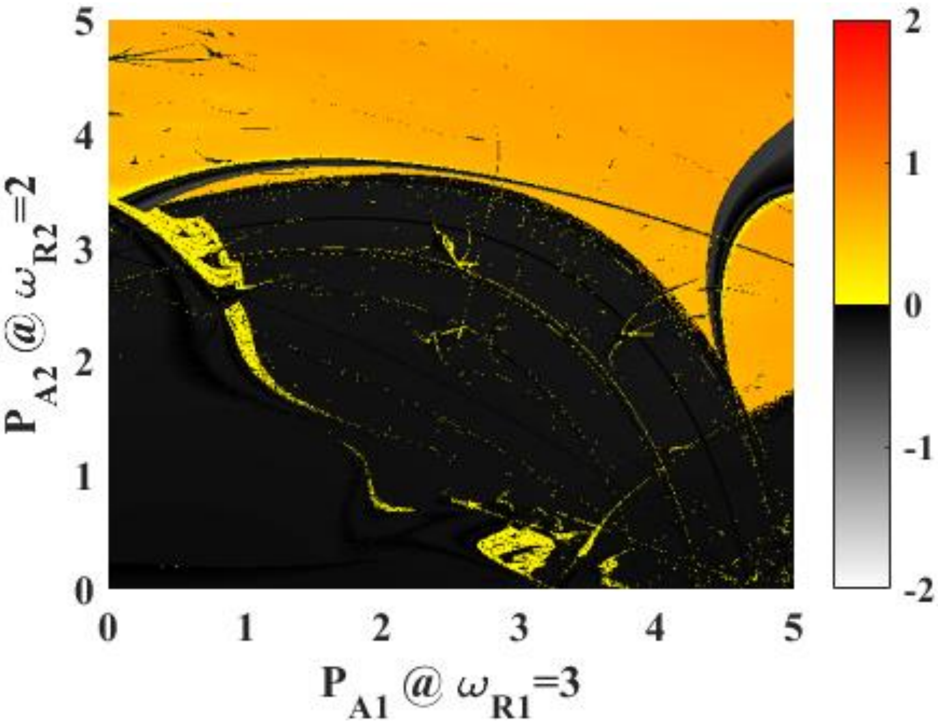



- *Period*

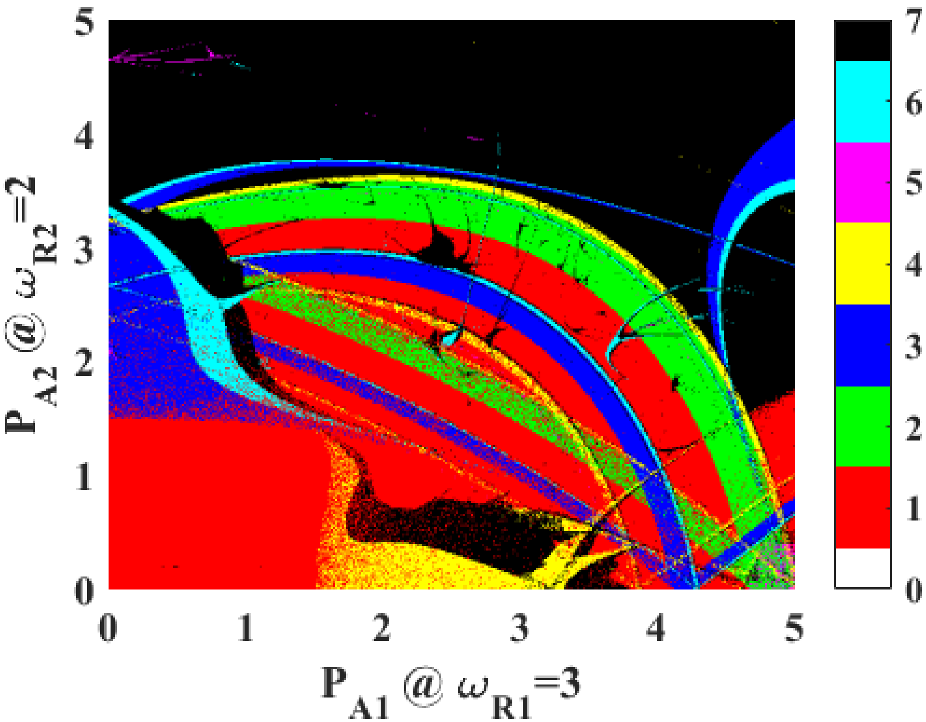


# High Resolution Plots

- *Lyapunov exponent*

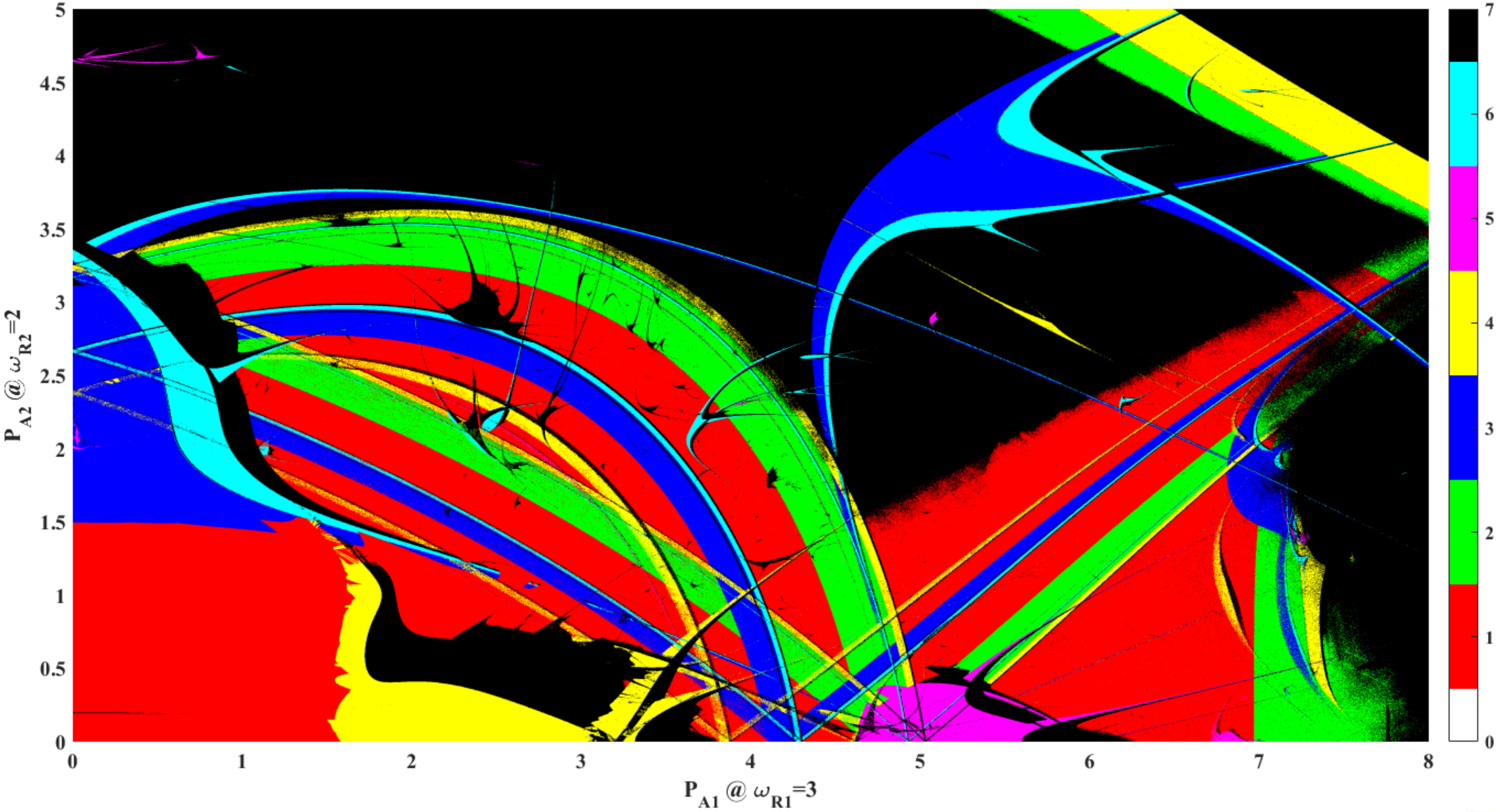



- *Period*



# High Resolution Plots

- *Resolution: 2500 X 8000, IC: 25, Problem size: 250 million*



# Thank you for your attention!