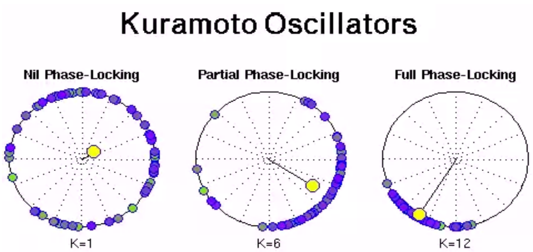
Critical synchronization dynamics of the Kuramoto model on connectome and small world graphs

Géza Ódor MTA-EK-MFA Complex Systems, Budapest
Jeffrey Kelling HZDR, Dresden

Theoretical research and experiments suggest that the brain operates at or near a **critical state** between sustained activity and an inactive phase, exhibiting optimal computational properties (see: *PRL 110, 178101 (2013)*)

Individual neurons emit periodic signals *(PNAS 113 (2016) 3341)* → criticality at the **synchronization transition point**

Kuramoto oscillator model (1975)



Nil, partial and full phase-locking in an all-to-all network of Kuramoto oscillators. Phase-locking is governed by the coupling strength K and the distribution of intrinisic frequencies ω. Here, the intrinsic frequencies were drawn from a normal distribution (M=0.5Hz, SD=0.5Hz). The yellow disk marks the phase centroid. Its radius is a measure of coherence.

$$\dot{\theta}_i(t) = \omega_{i,0} + \frac{K}{k_i} \sum_j W_{ij} \sin[\theta_i(t) - \theta_j(t)]$$

phases $\theta_i(t)$ in-degrees k_i

global coupling K is the control parameter

weighted adjacency matrix W_{ij}

$\omega_{i,0}$ is the intrinsic frequency of the i-th oscillator,

Order parameter : average phase:

$$R(t) = \frac{1}{N} \left| \sum_{j=1}^{N} e^{i\theta_j(t)} \right|$$

Non-zero, above critical coupling strength $K > K_c$,
tends to zero for $K \leq K_c$ as $R \propto (1/N)^{1/2}$
or exhibits an initial growth: $R(t,N) = N^{-1/2} t^{\eta} f_{\uparrow}(t/N^{\bar{z}})$ *for incoherent initial state*

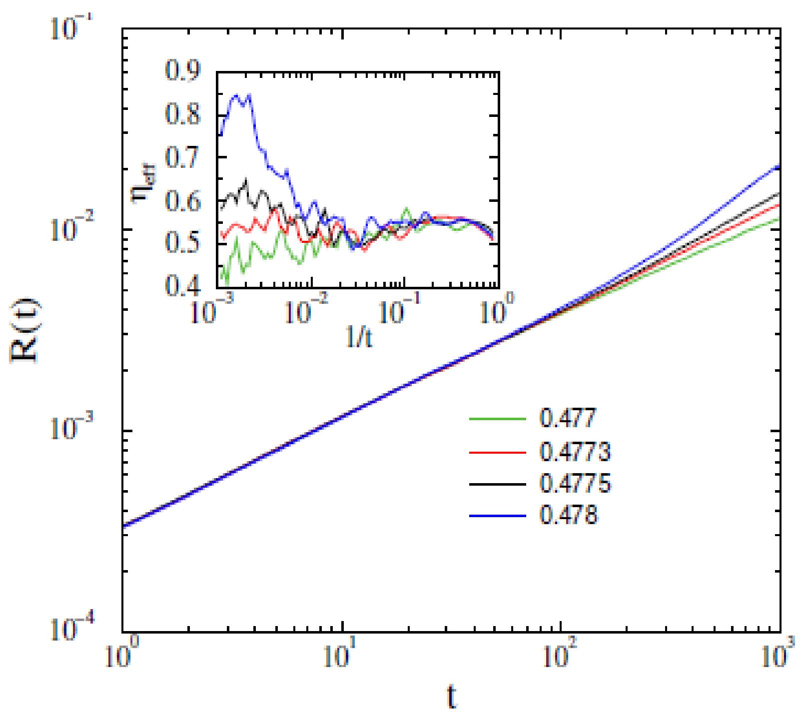
Critical synchronization transition for D > 4 spatial dimensions,
which is mean-field like: i.e. $D \to \infty$ (full graph)

The dynamical behavior suffers very strong corrections to scaling and chaoticity, see:

Róbert Juhász, Jeffrey Kelling and Géza Ódor:
Critical dynamics of the Kuramoto model on sparse random networks
J. Stat. Mech. (2019) 053403

Test of Kuramoto model on sparse synthetic networks

2D lattices of linear size $L = 6000$,
periodic boundary conditions,
+ extra random long link between
connecting any edges: $<k> = 5$,
90,000,000 edges



Growth runs from random initial state
Runge-Kutta-4 parallelized for GPUs
Maximum time: $t_{max} = 1000$,
average over: *10000* independent ω_i realizations
Critical point located at $K = 0.4773$
Critical exponent: $\eta = 0.55\ (10)$

Determination of desynchroniztion avalanche exponent τ_t

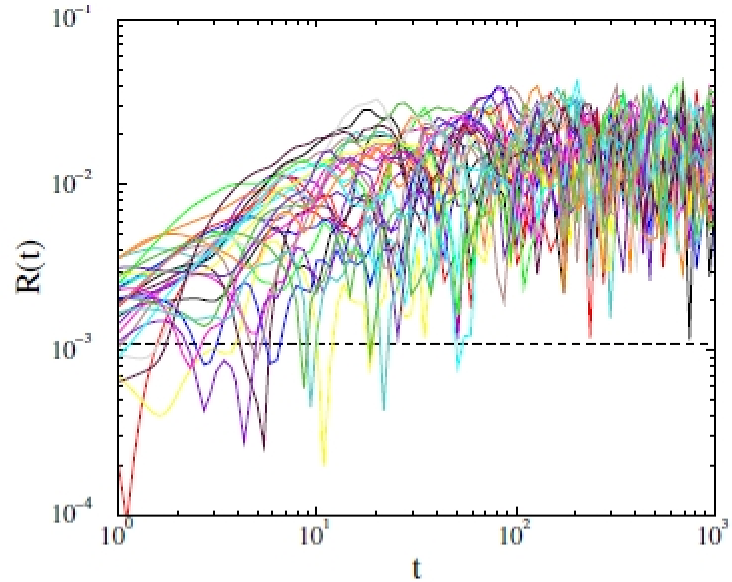
Measure characteristic times t_x of first dip below: $R_c = (1/N)^{1/2}$

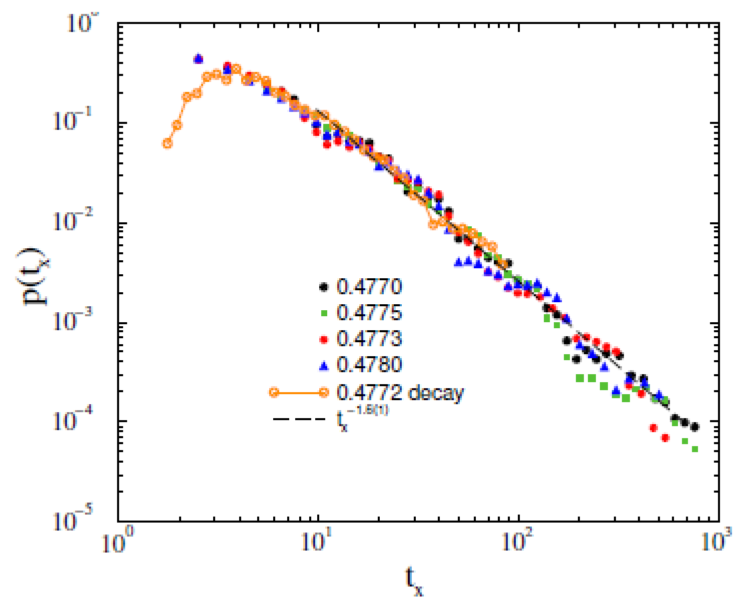
average over: *10,000* independent ω_i realizations

Histogramming of t_x at the critical point

Critical exponent: $\tau_t = 1.6\ (1)$

obtained by fitting for the PL tails

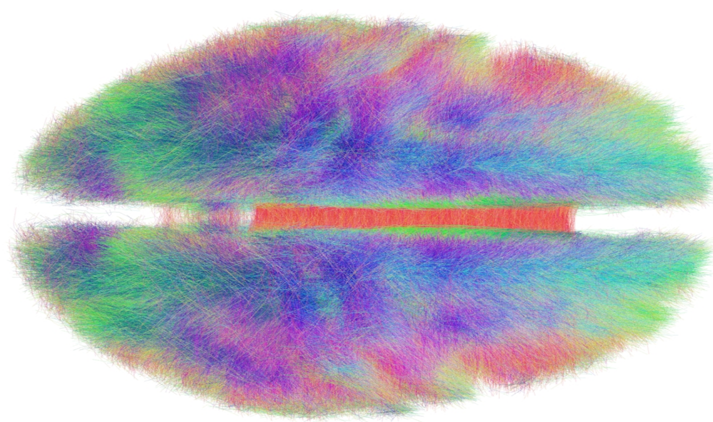




What do we know about neuron networks ?

The largest precisely explored structural networks contains ~302 neurons (C. Elegans) (very recently fruit fly is reported)

Connectomes, obtained by approximative methods like diffusion MRI contain < 10^6 nodes (voxels)



Recently DMRI tractrography was confirmed by tract-tracing in ferret

Open Connectome Large Human graphs

Diffusion and structural MRI images with
1 mm^3 voxel resolution :
10 ^{5} *–10* ^{6} nodes

Hierarchical modular graphs

Top level: *70* brain region (Desikan atlas)

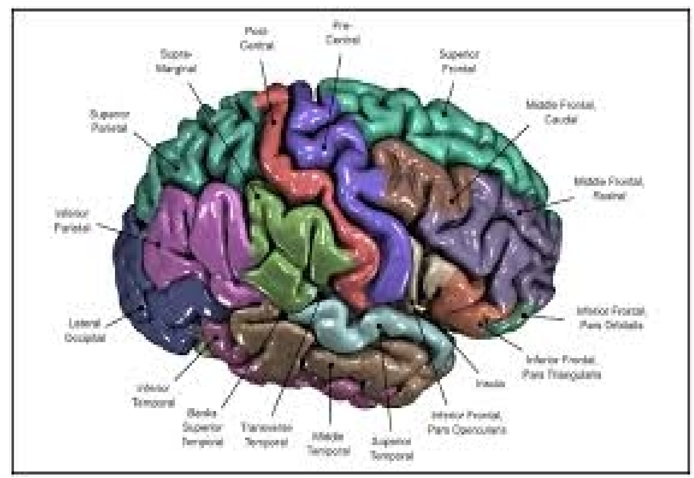
Lower levels: Deterministic tractography:
Fiber Assignment by Continuous Tracking (FACT) algorithm

Map : voxel → vertex (~ 10^7)

fiber → edge (~ 10^{10})

+ noise reduction → graph

undirected, weighted



SCIENTIFIC REPORTS

OPEN

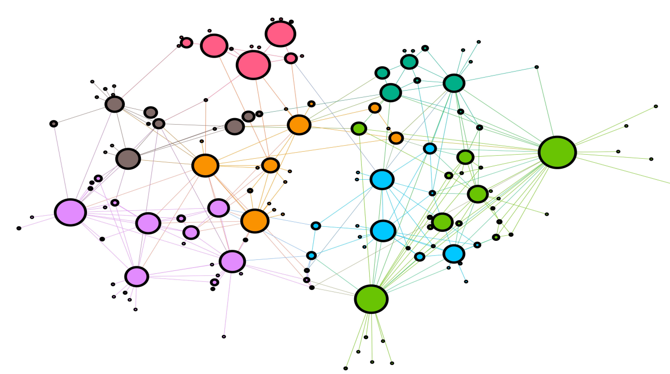
The topology of large Open Connectome networks for the human brain

Received: 18 December 2015
Accepted: 12 May 2016
Published: 07 June 2016

Michael T. Gastner[1,2] & Géza Ódor[2]

The structural human connectome (i.e. the network of fiber connections in the brain) can be analyzed at ever finer spatial resolution thanks to advances in neuroimaging. Here we analyze several large data sets for the human brain network made available by the Open Connectome Project. We apply statistical model selection to characterize the degree distributions of graphs containing up to $\simeq 10^6$ nodes and $\simeq 10^8$ edges. A three-parameter generalized Weibull (also known as a stretched exponential) distribution is a good fit to most of the observed degree distributions. For almost all networks, simple power laws cannot fit the data, but in some cases there is statistical support for power laws with an exponential cutoff. We also calculate the topological (graph) dimension D and the small-world coefficient σ of these networks. While σ suggests a small-world topology, we found that $D < 4$ showing that long-distance connections provide only a small correction to the topology of the embedding three-dimensional space.

Small world, still finite dimensional,
non-scale free,
universal modular graphs



Kuramoto solution for the KKI-18 graph with
N= *836 733* nodes and *41 523 931* weigthed edges

The synchronization transition point determined by growth as before

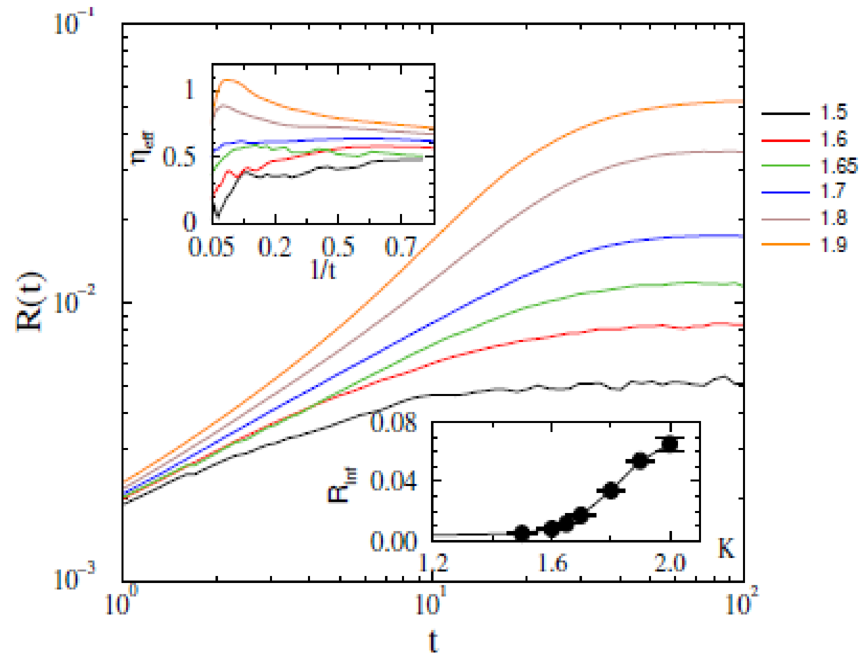
KKI-18 has $D = 3.05 < 4$ →

No real phase transition, crossover

Due to the fat-tailed link weight distribution, incoming weight normalization is applied:

$$W'_{i,j} = W_{i,j} / \sum_{j \in \text{neighb.of } i} W_{i,j}$$

$K_c = 1.7$ and growth exponent: $\eta = 0.6(1)$



Duration distribution for the KKI-18 graph

Measure characteristic times t_x of first

dip below: $R_c = (1/N)^{1/2}$

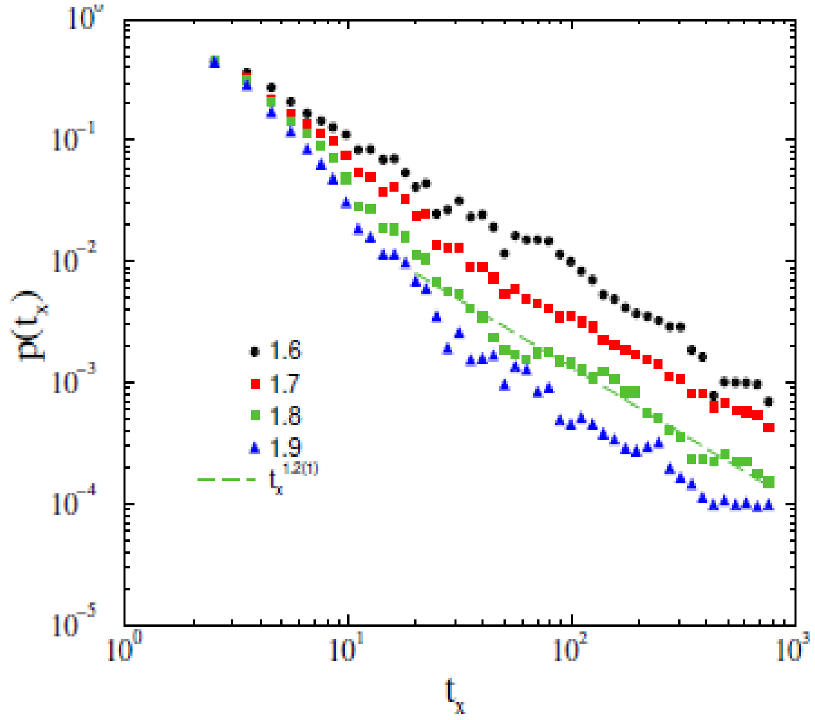
average over: *10.000* independent ω_i realizations

Histogramming of t_x at the critical point

Critical exponent: $\tau_t = 1.2\ (1)$

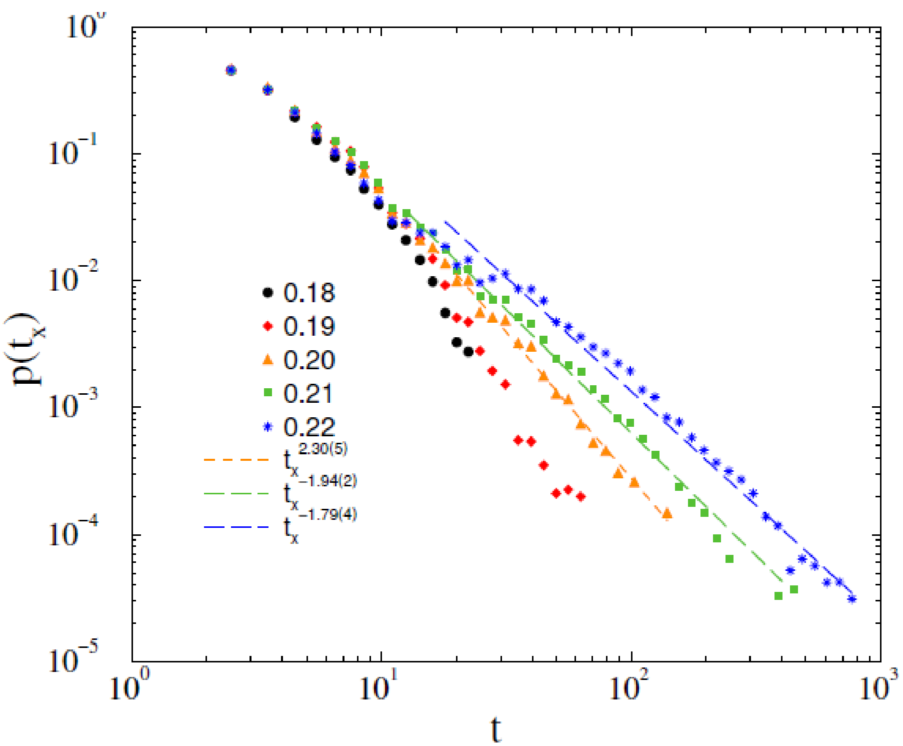
obtained by fitting for the PL tails

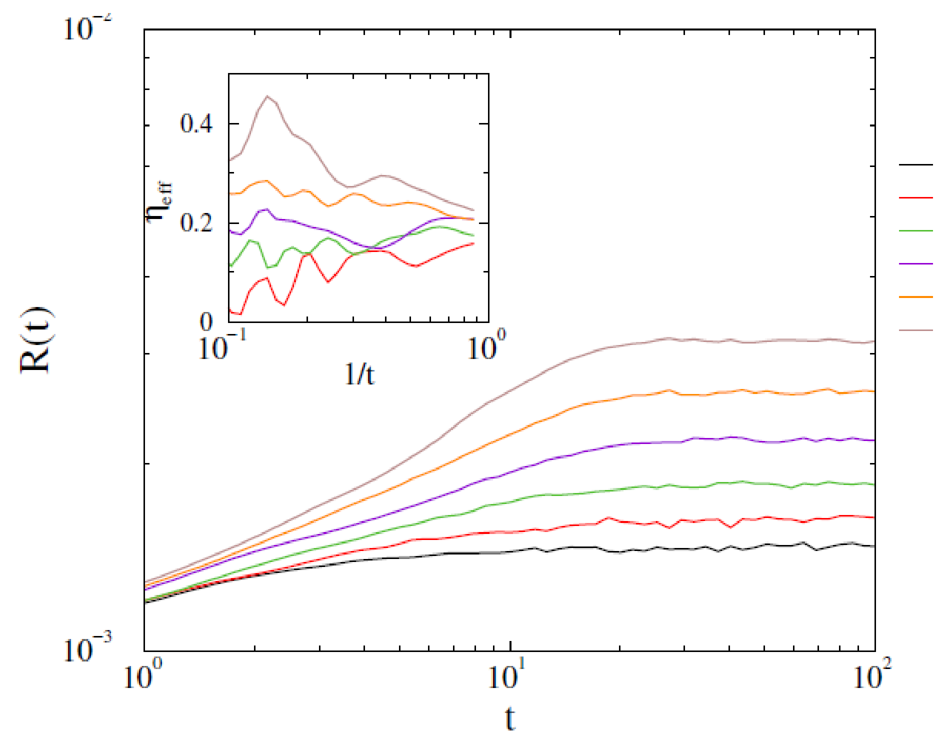
Out of range of experiments :
$1.5 < \tau_t < 2.4$ (*Palva et al 2013)*



Inhibitory (negative) links compared to experiments

Inhibitions: 5-20% of nodes: $w_{ij} \rightarrow -w_{ij}$ randomly





Scaling exponent within experimental range: 1.5 < τ_t < 2.4

J.M. Palva et al PNAS 110 (2013) 3585

K dependent scaling exponents: Frustrated synchronization ?

Conclusions

Heterogeneity effects are considered on large human connectomes and 2d + long range lattices of extremely large sizes

Kuramoto synchronziation equation is solved by 4th order Runge-Kutte method, implemented on parallel **GPU-s**

In case of 2d + long range lattices we determined the temporal mean-field like solution, with very strong corrections to scaling

De-synchronization characteristic exponent is found: $\tau_t = 1.6\,(1)$

On the normalized, weighted ***KKI-18*** graph, describing variable node sensitivity, $\tau_t = 1.2(1)$ *,* out of experimental range

On the normalized, inhibitory ***KKI-18*** graph $\tau_t = 1.8(2)$ *,* within experimental range

Frustrated synchronization sub-critically !

Insensitivity to 5 - 20% link sign reversal, robustness

Details : G. Ódor and J. Kelling: arXiv:1903.00385,
R. Juhasz, G. O, J. K : J. Stat. Mech. (2019) 053403

HPC support from Hungaian HPC infra: KIFU is acknowledged