Laboratory observation of water surface polygon vortices

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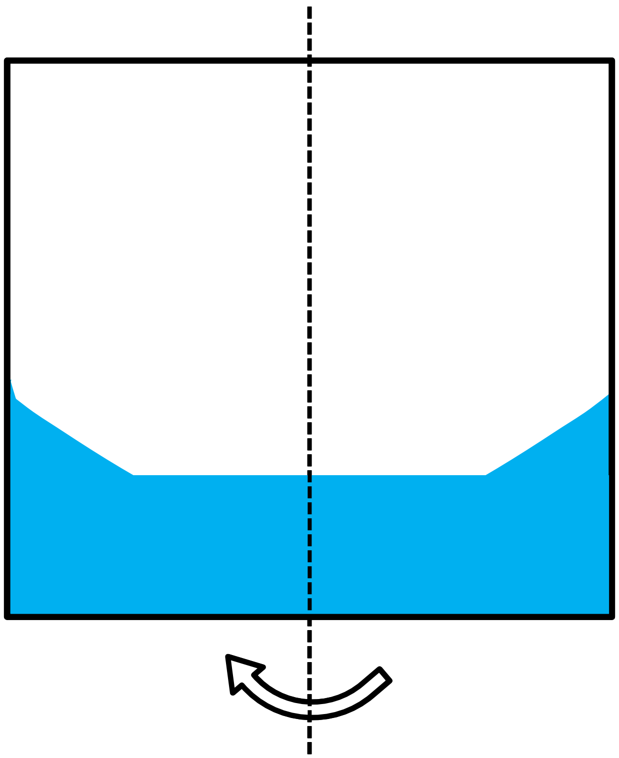
Outline of the presentation

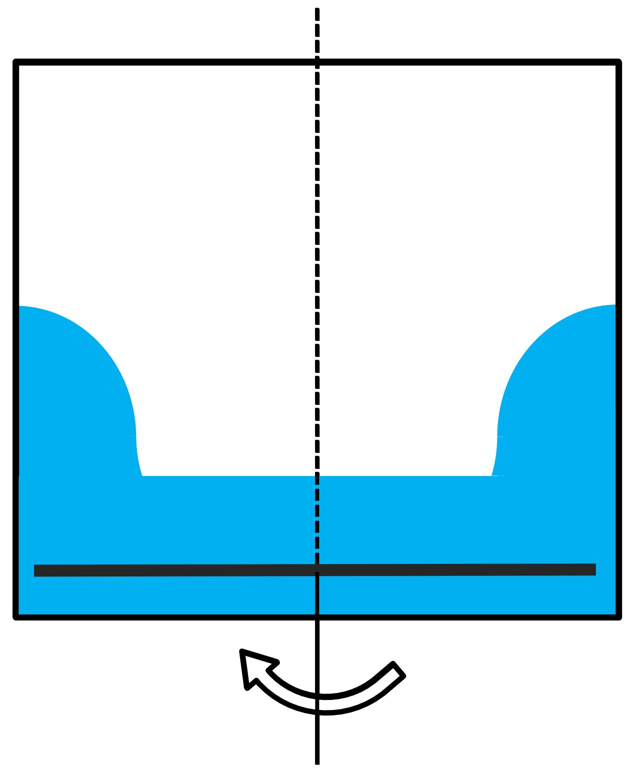
I. Introduction

I.1. Newton’s bucket – polygon vortex

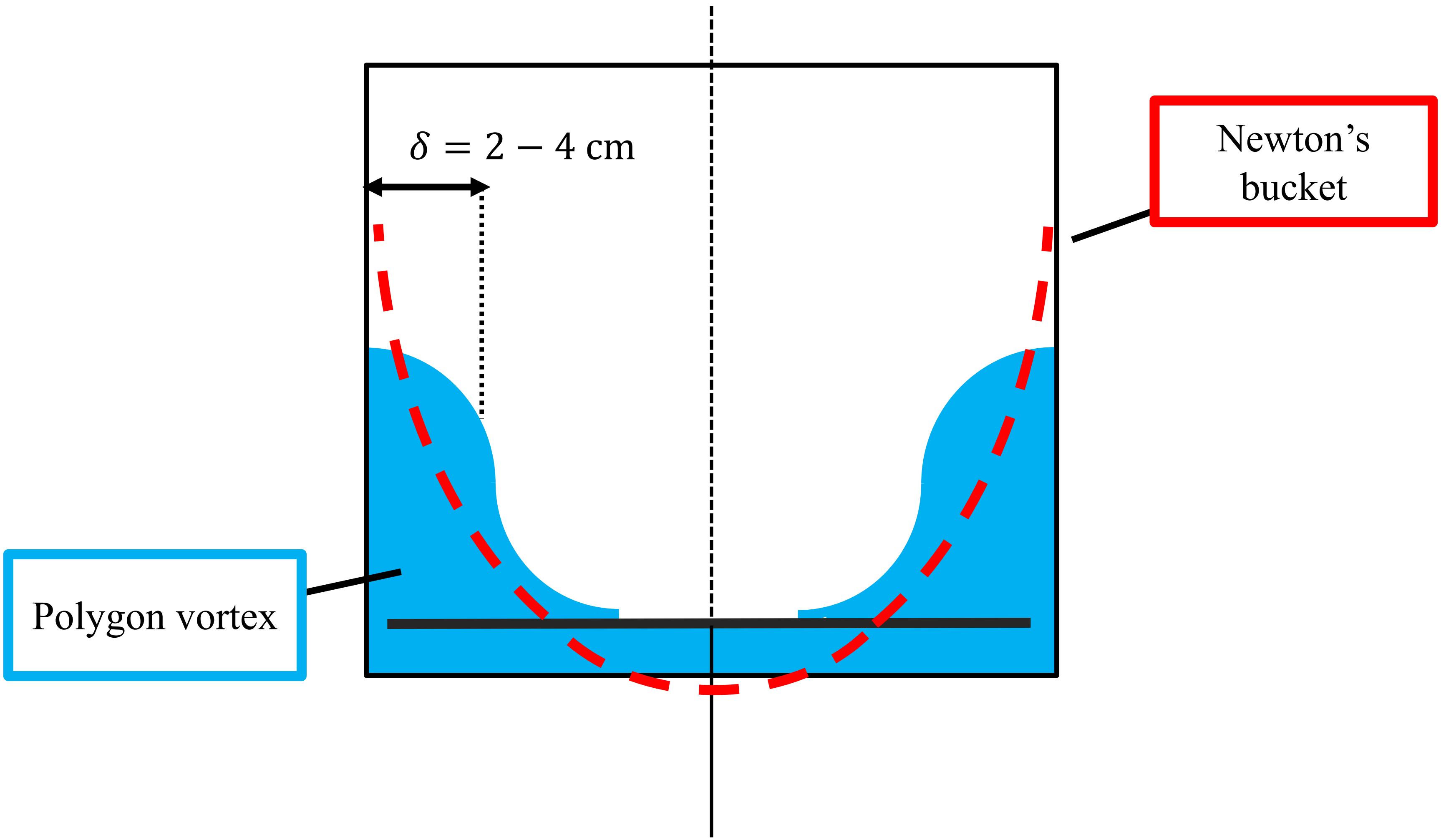
Newton’s bucket



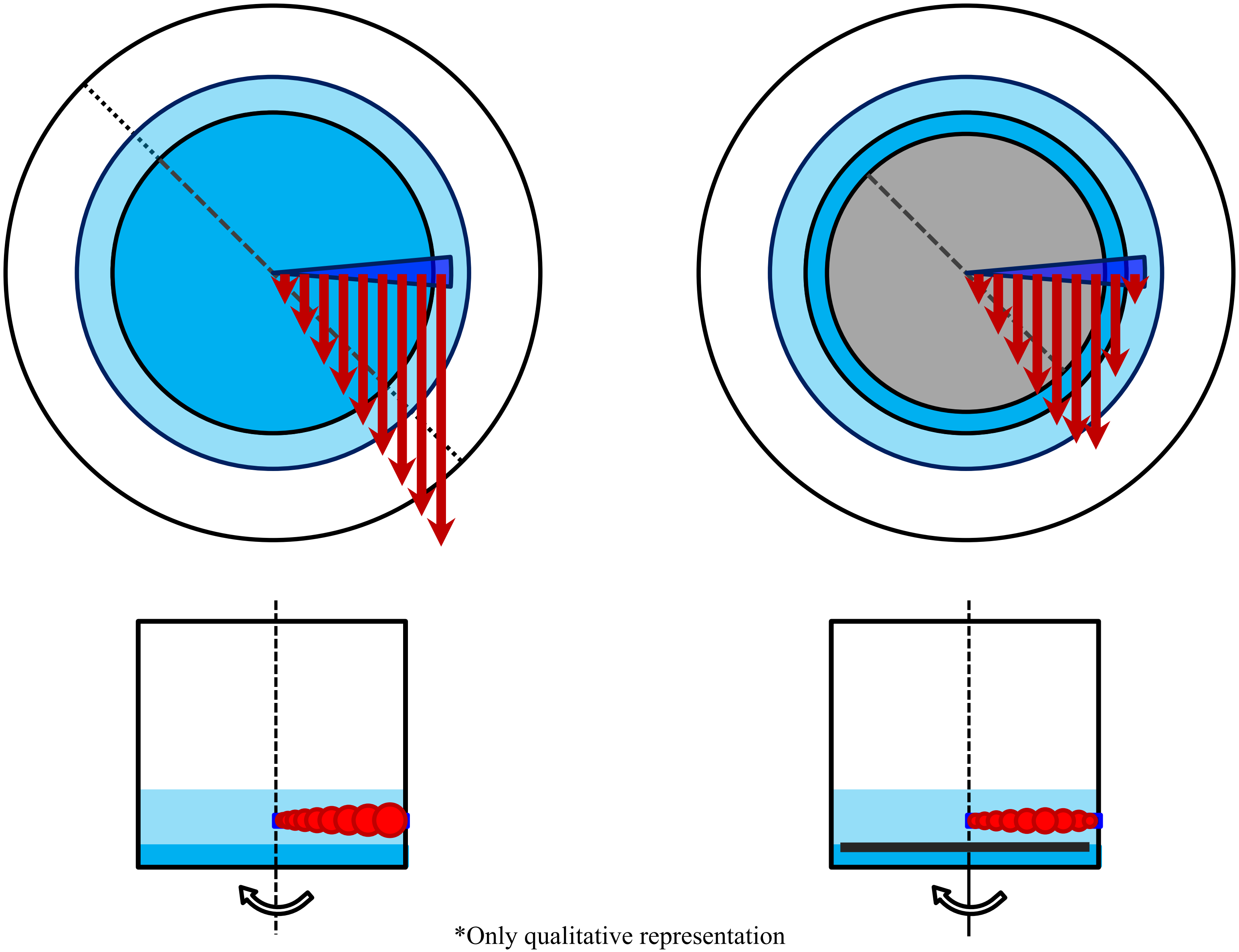
Polygon vortex



I.1. Newton’s bucket – polygon vortex

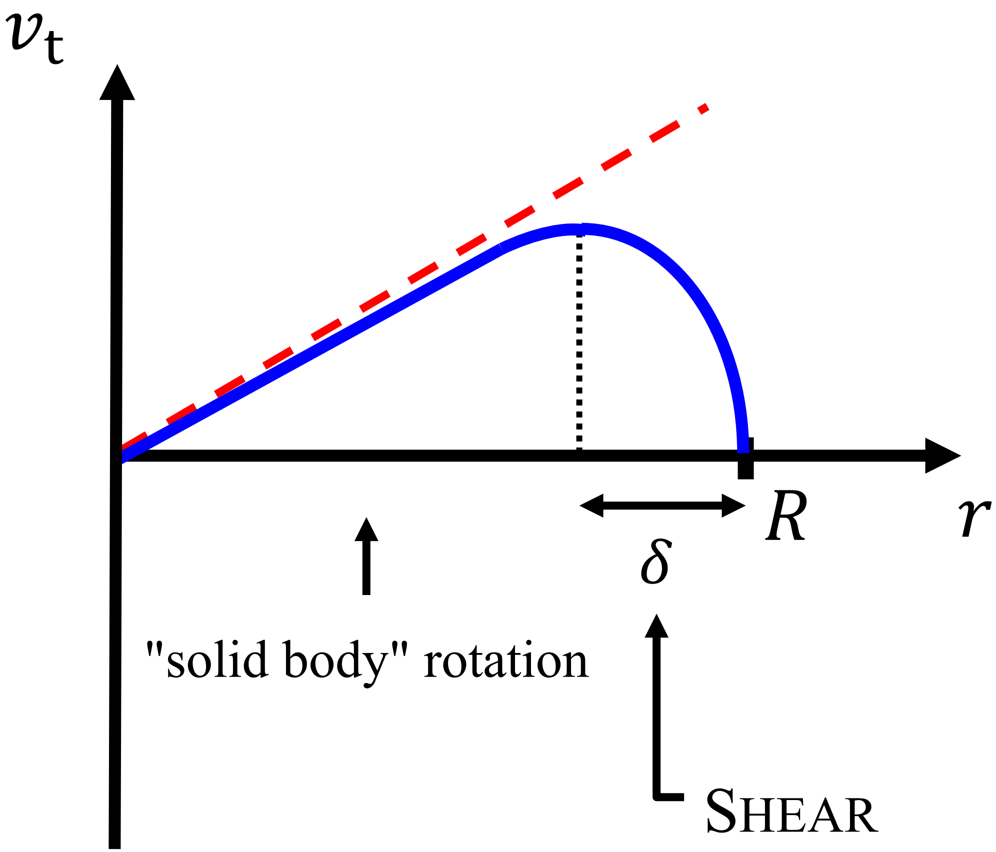


I.1. Newton's bucket – polygon vortex

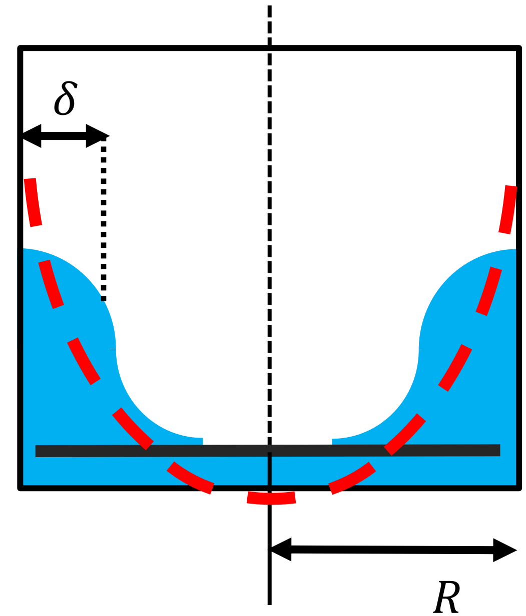


*Only qualitative representation

I.1. Newton's bucket – polygon vortex

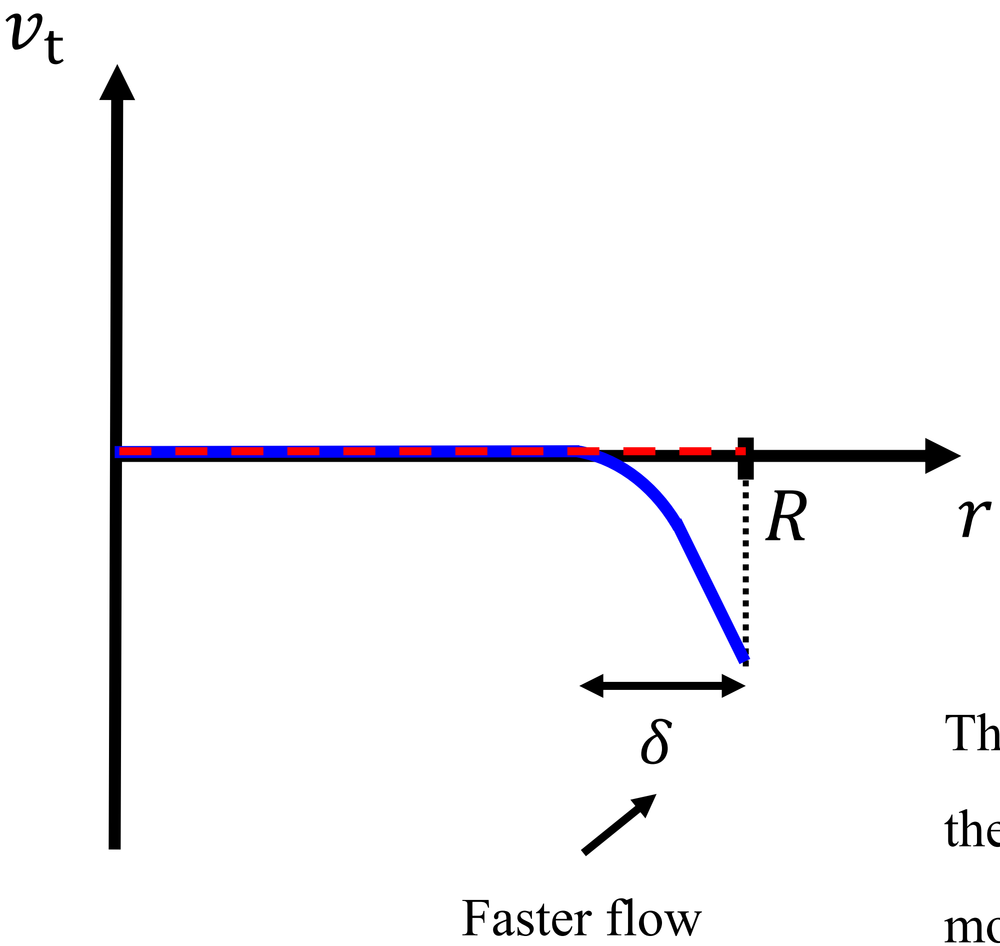


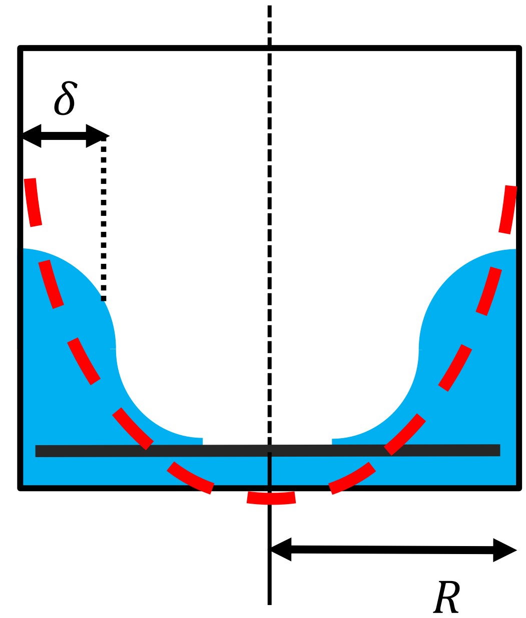
From the laboratory frame



I.1. Newton's bucket – polygon vortex

From the co-rotating frame

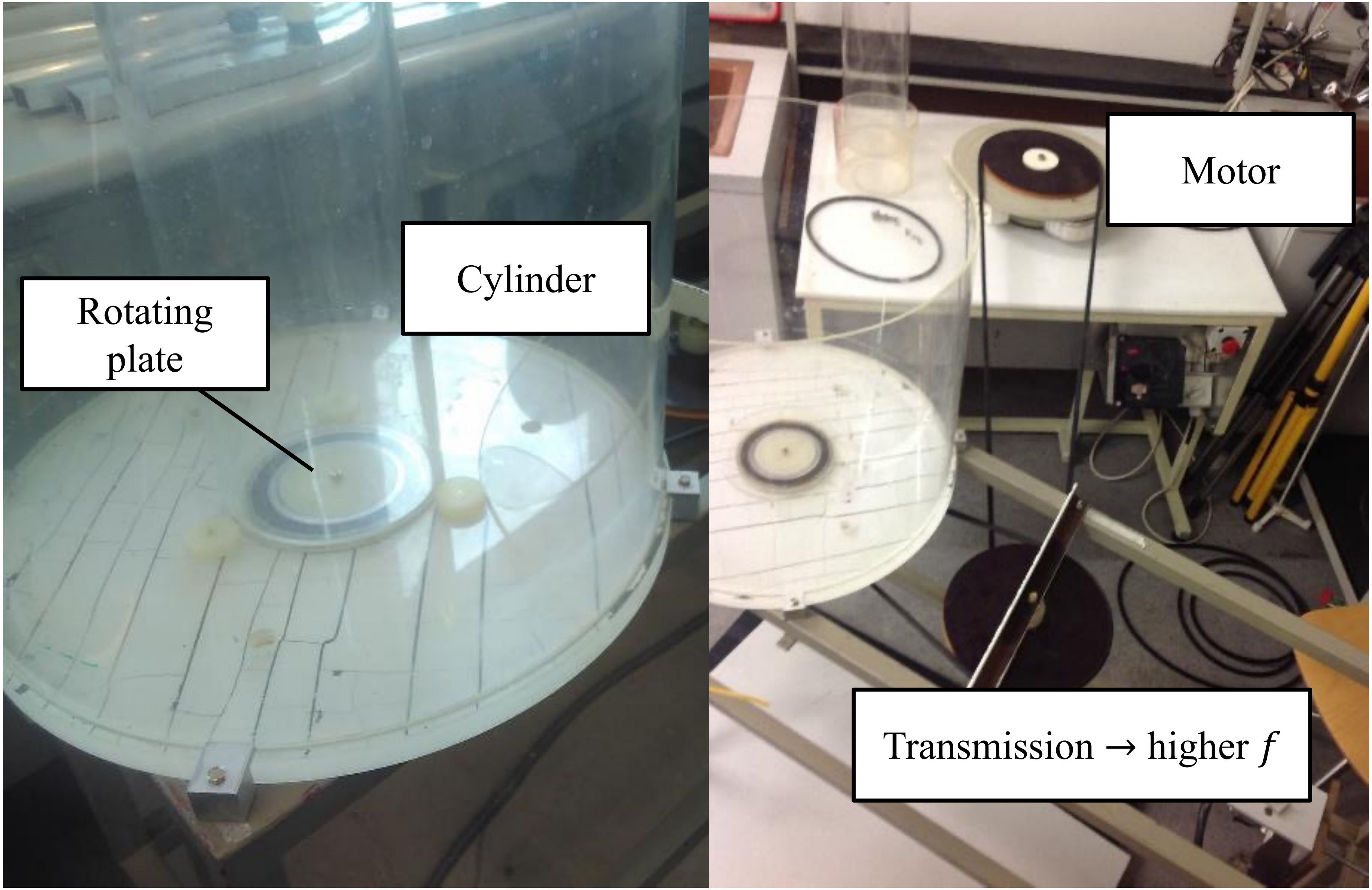




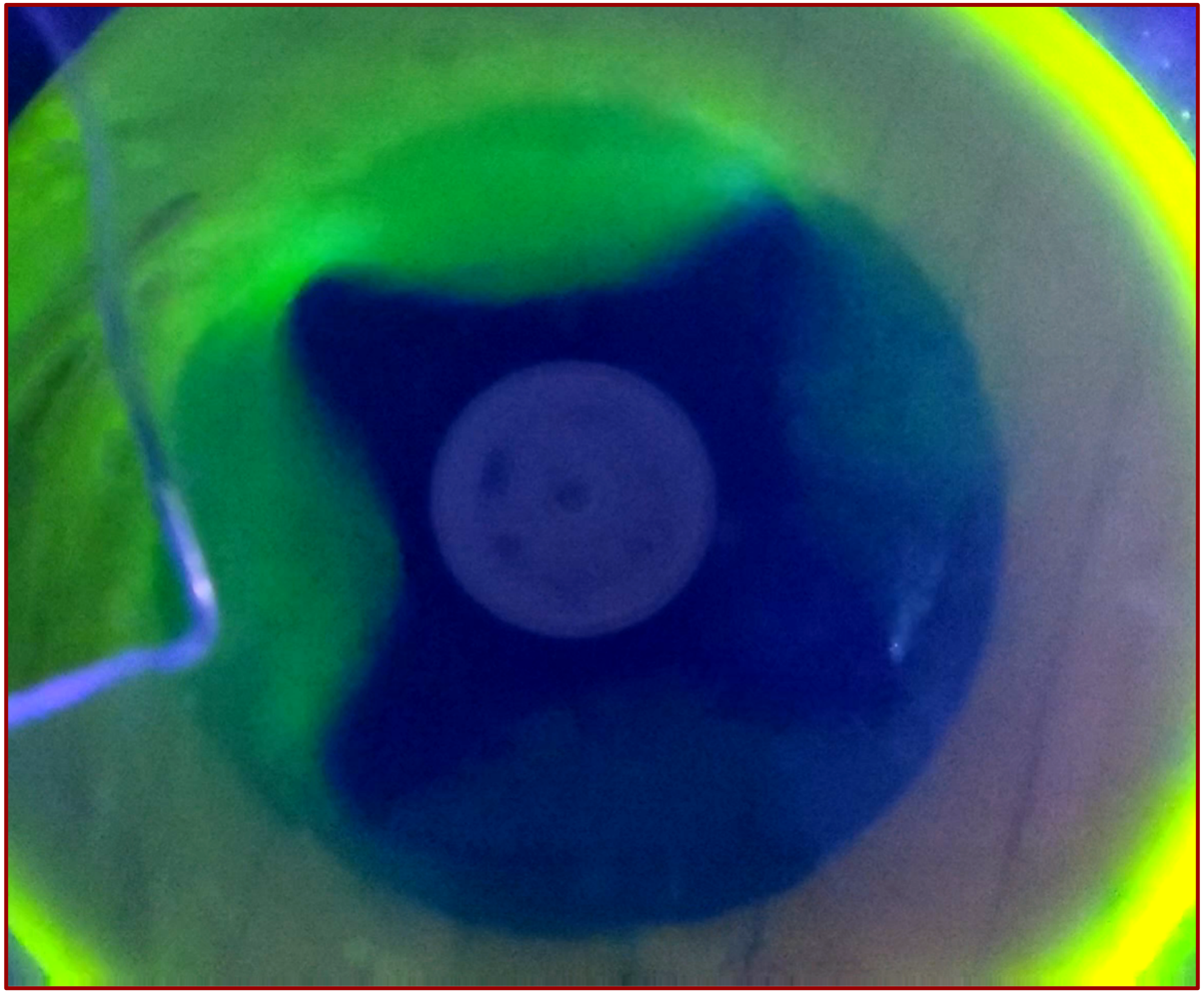
This δ is the reason that instead of the simple Newton's bucket we see more complex shapes.

II. Reproduction of the phenomenon

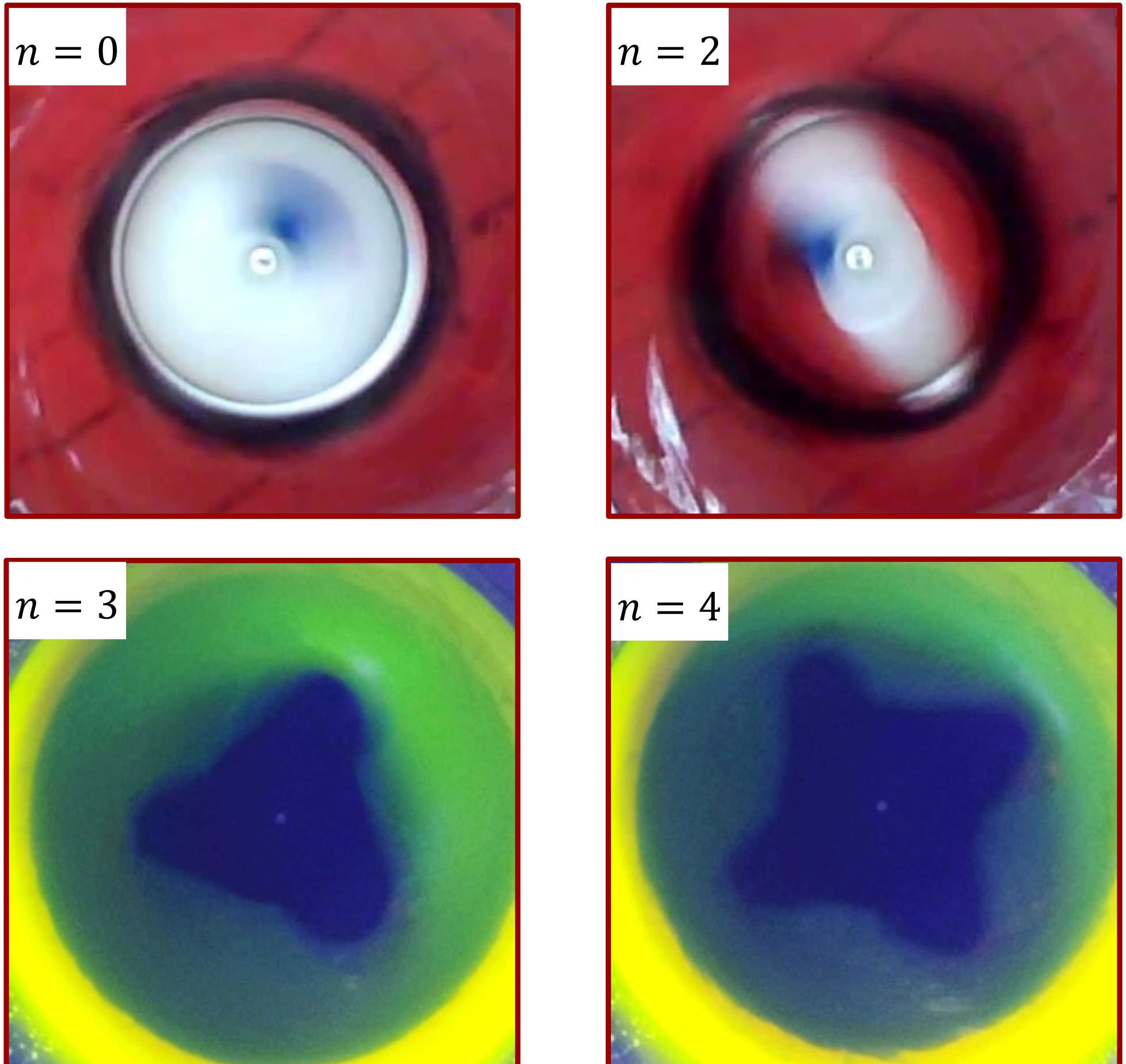
II.1. Measurement setup



II.2. The phenomenon



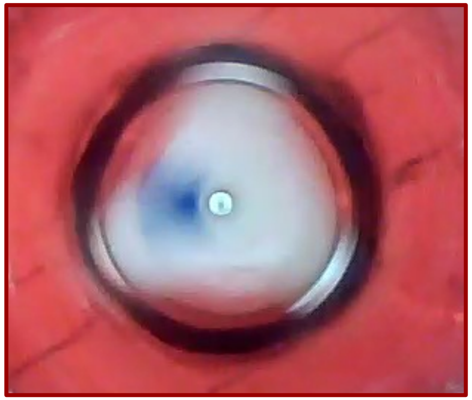
II.2. The phenomenon

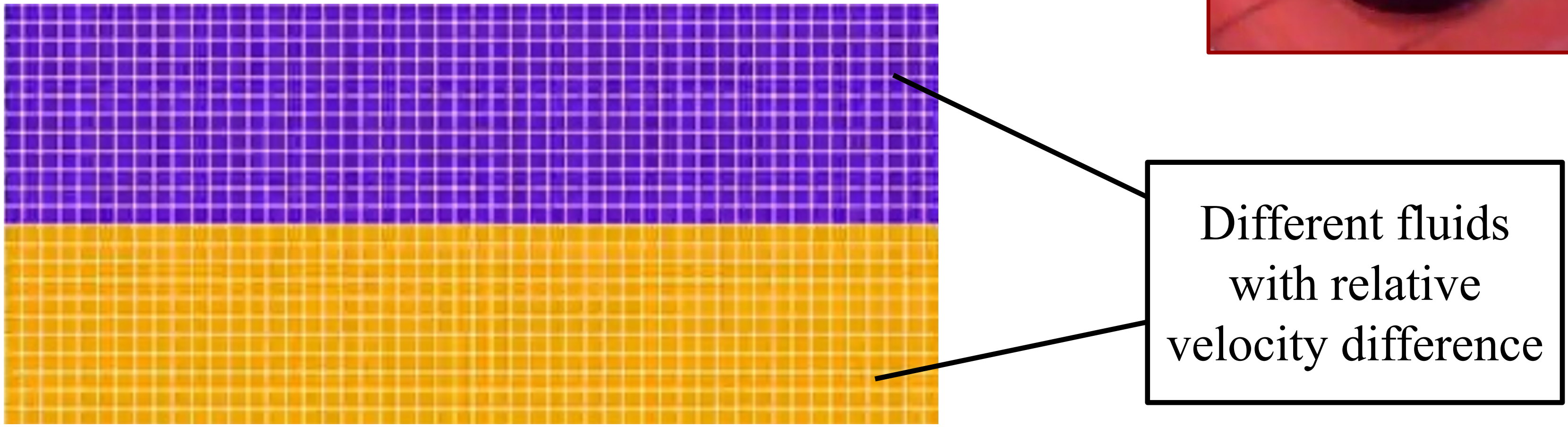


III. Qualitative explanation

III.1. Hydrodynamic instability

- **What do we see?**
  - Hydrodynamic instability
  - An example: Kelvin-Helmholtz-instability



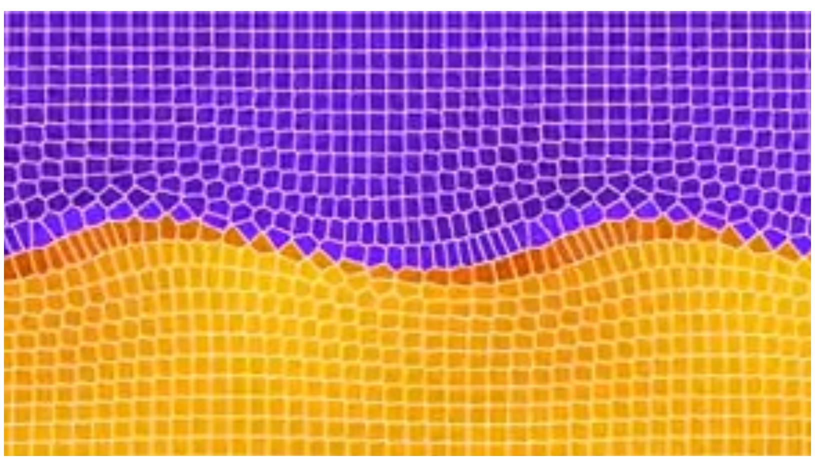


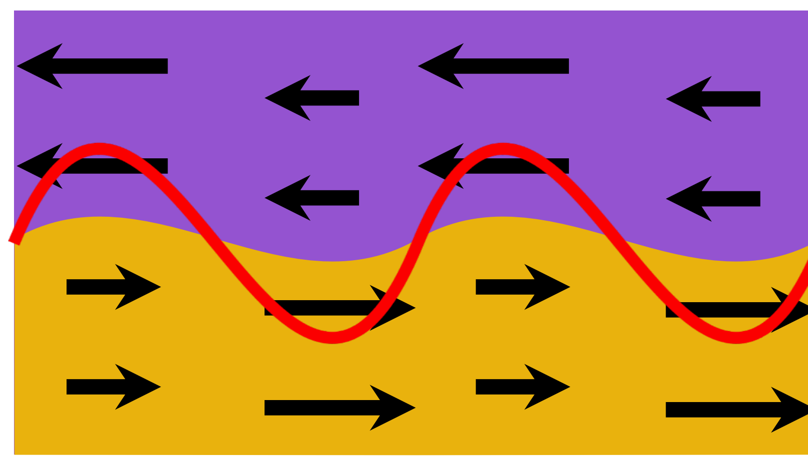
- In our case:
  - No different fluids, only relative velocity difference → velocity shear (δ)

Kevin Schaal. „Kelvin-Helmholtz instability". *Youtube, 2012. Nov. 26., 1:20. https://www.youtube.com/watch?v=nuK9PvlpUNg*

III.1. Hydrodynamic instability

Velocity shear in the fluid → Perturbations form → Bernoulli's principle → These perturbations grow → Formation of wave-like shapes

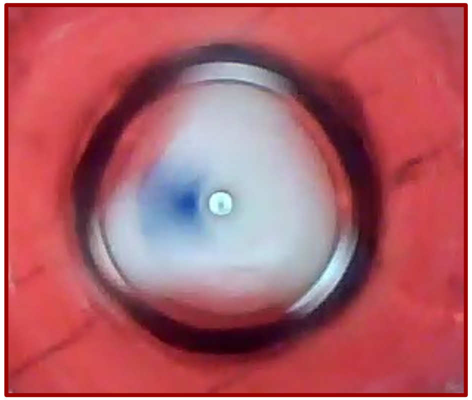




Kevin Schaal. „Kelvin-Helmholtz instability". *Youtube, 2012. Nov. 26., 1:20. https://www.youtube.com/watch?v=nuK9PvlpUNg*

III.1. Hydrodynamic instability

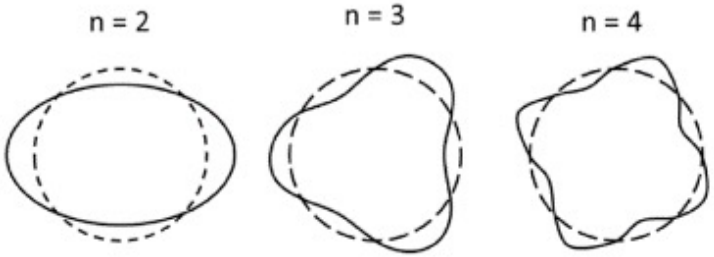
In a nutshell:
Waves interfere with themselves
↓
Superposition



Similar waves form

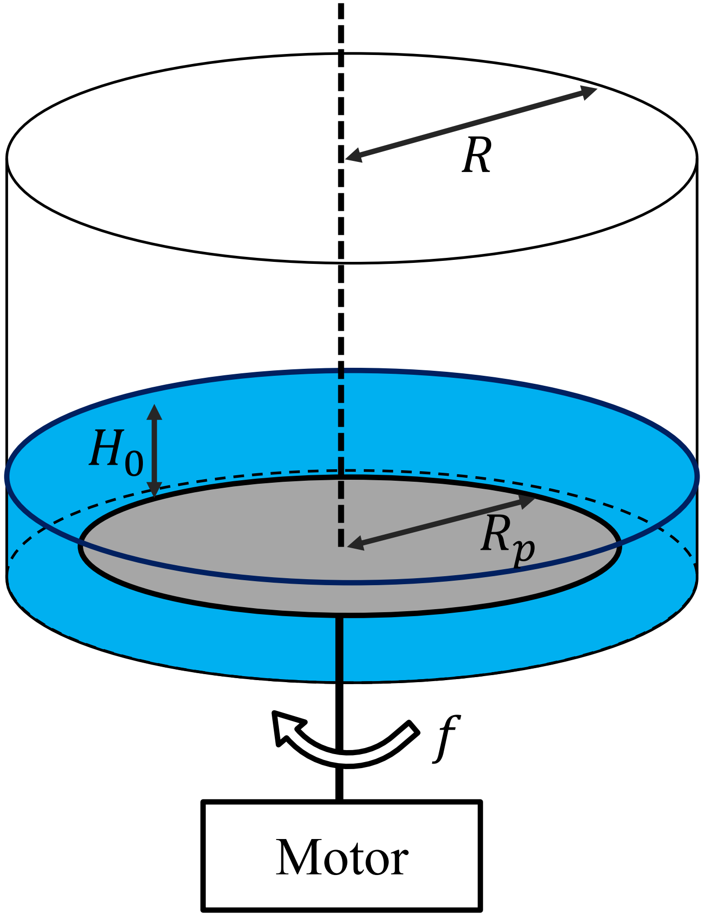
BUT! We have a periodic boundary condition

Only those wavelengths are amplified, which fit on this loop $n \in \mathbb{N}$ times



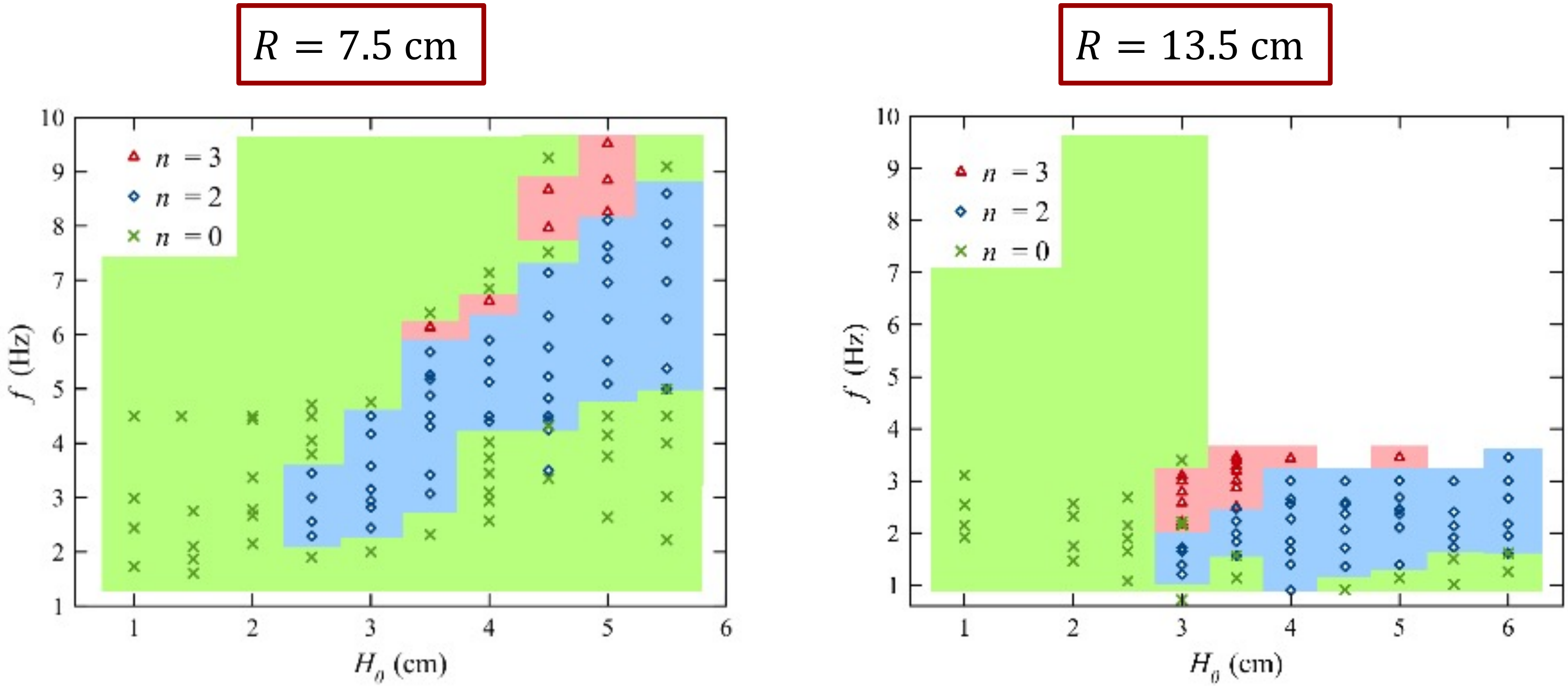
IV. Relevant parameters

IV.1. Relevant parameters



Relevant parameters	
Radius of the cylinder	R
Radius of the plate	R_p
Starting height of the water	H_0
Frequency of the rotation	f
Density	ρ
Kinematic viscosity	ν
Height of water below the plate	

IV.2. Phase diagram



- Conclusions
  - Higher radius → we get polygon-vortices „sooner”
  - Higher water level → lower n
  - Higher frequency → higher n
  - Bottom and top limits

V. Quantitative theory

V.1. Equation of motion

- Navier-Stokes equation:

$$\rho\left[\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right] = \underbrace{-\nabla p}_{\text{Inner pressure differences}} + \underbrace{\mu\nabla^2\boldsymbol{u}}_{\text{Viscosity}} + \underbrace{\boldsymbol{g}}_{\text{External force}}$$

- Equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{u}) = 0$$

- Reynolds number (in our measurements)

$$\mathrm{Re} = \frac{\omega R^2}{\nu}$$

$$100\,000 \leq \mathrm{Re} \leq 10\,000\,000$$

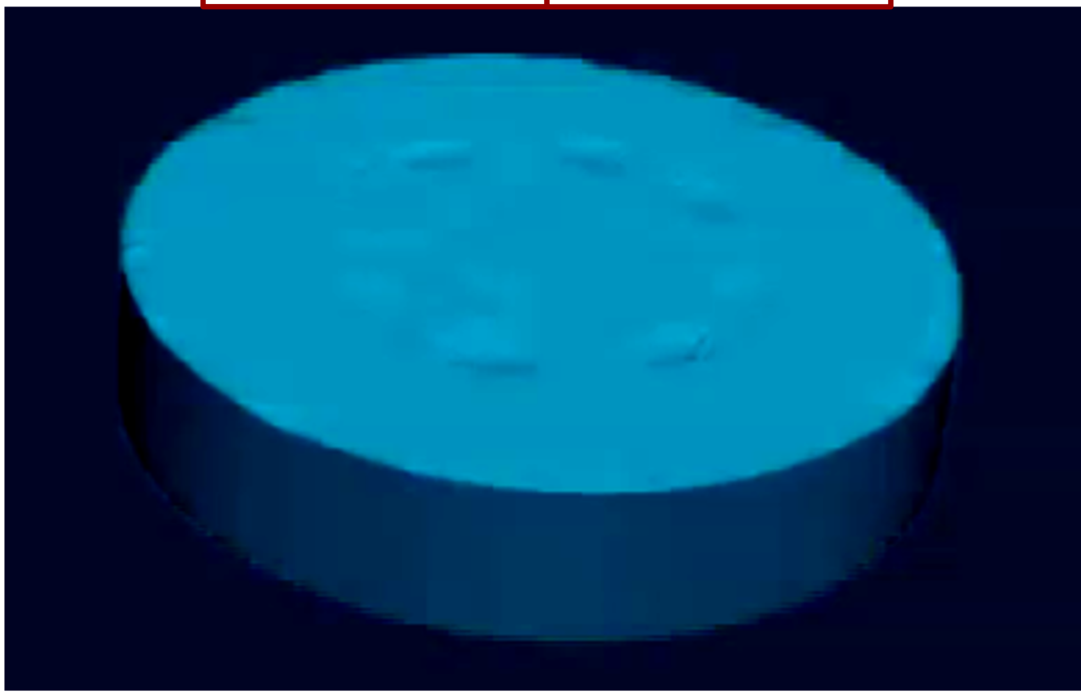
V.2. Pilot simulations

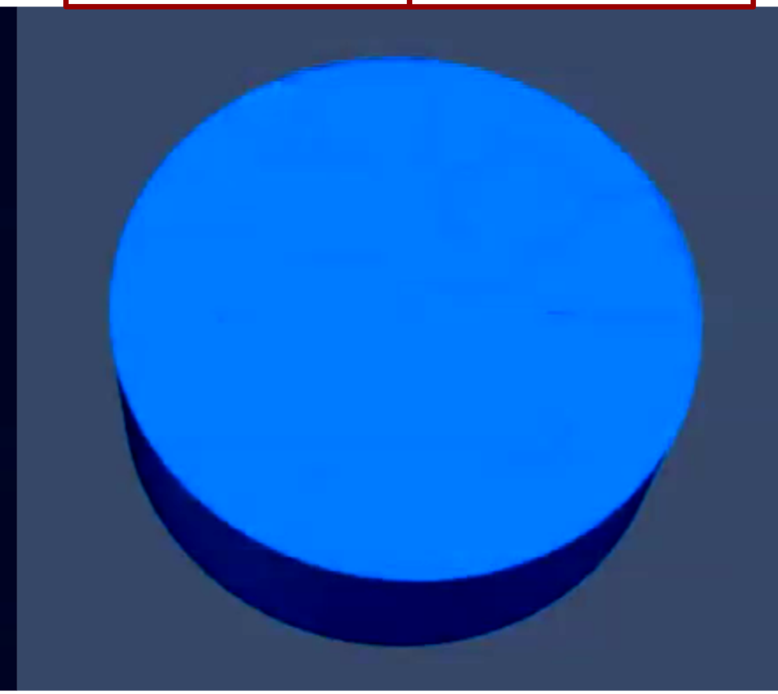
$H_0 = 1$ cm

$R = 2.5$ cm | $f = 5$ Hz

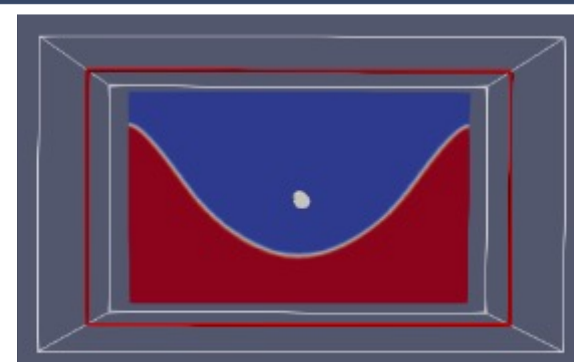
$H_0 = 5$ cm

$R = 7.5$ cm | $f = 9$ Hz





- The calculation takes days to compute
- GPU lab of Wigner Research Centre of Physics
  - 48 CPU core; 760 GB RAM
  - 16 CPU core; 16 GB RAM

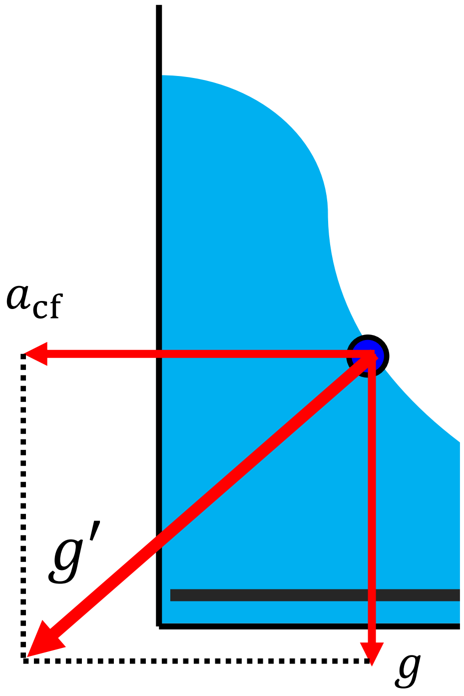


V.2. Pilot simulations

- Numerical simulations (Navier-Stokes)
  - Very complex:
    - Complex boundary conditions
    - Dynamic 3D system (very high time and spatial resolution needed)
    - Every single point on a phase diagram would need to be simulated
    - Even pilot simulations required days to calculate
  - Only a few articles touch this area (e.g.)
    - R. BERGMANN, L. TOPHØJ, T. A. M. HOMAN, P. HERSEN, A. ANDERSEN, & T. BOHR. 2011 Polygon formation and surface flow on a rotating fluid surface. J. Fluid Mech. **679**, 415–431.
    - L. TOPHØJ, J. MOUGEL, T. BOHR., D. FABRE. 2013 Rotating Polygon Instability of a Swirling Free Surface Flow. PRL. **110**, 194502-1 - 194502-5.
- Generally with hydrodynamic instabilites the simulations are very complicated
  - Small perturbations grow immensely
  - Very detailed mesh required

V.3. „Tilted world”

From a co-rotating frame



$$g' = \sqrt{a_{\mathrm{cf}}^2 + g^2}$$

Polygon vortices:

Water surface waves on a tilted surface with an effective g'

V.4. Water surface wave dispersion relation

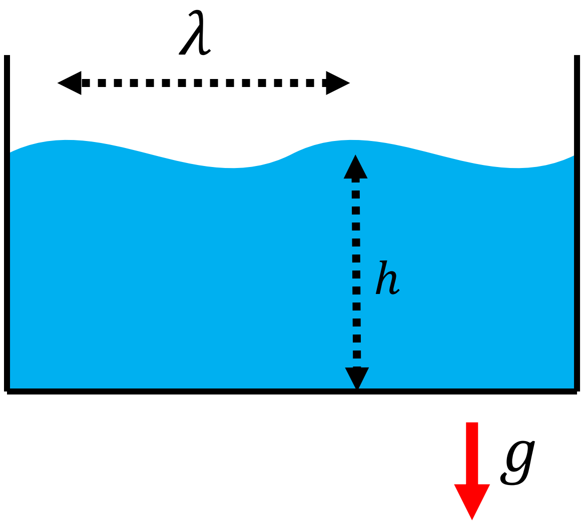
gravitational acceleration

$$f_{\mathrm{wave}} = \sqrt{\frac{g}{2\pi}\frac{1}{\lambda}\tanh\left(2\pi\frac{h}{\lambda}\right)}$$

water depth

frequency of the wave

wavelength



V.4. Water surface wave dispersion relation

In our case:

'effective' g

hard to determine

$h' \approx R - \bar{r}$ should

be a good estimate

for the order of

magnitude

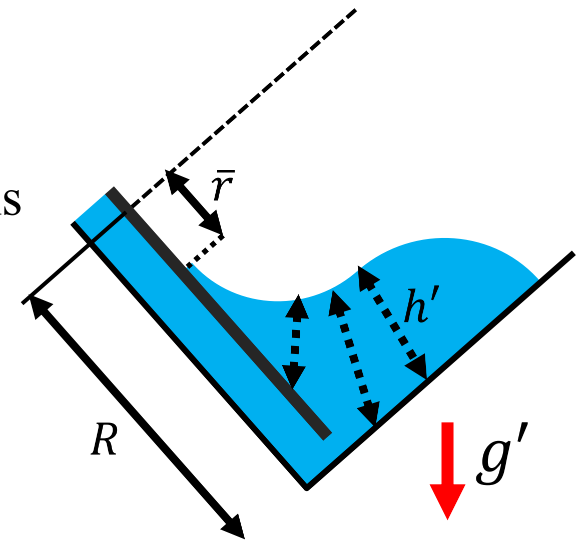
$$f_{\text{wave}} = \sqrt{\frac{g'}{2\pi}\frac{1}{\lambda}\tanh\left(2\pi\frac{h'}{\lambda}\right)}$$

frequency of the wave

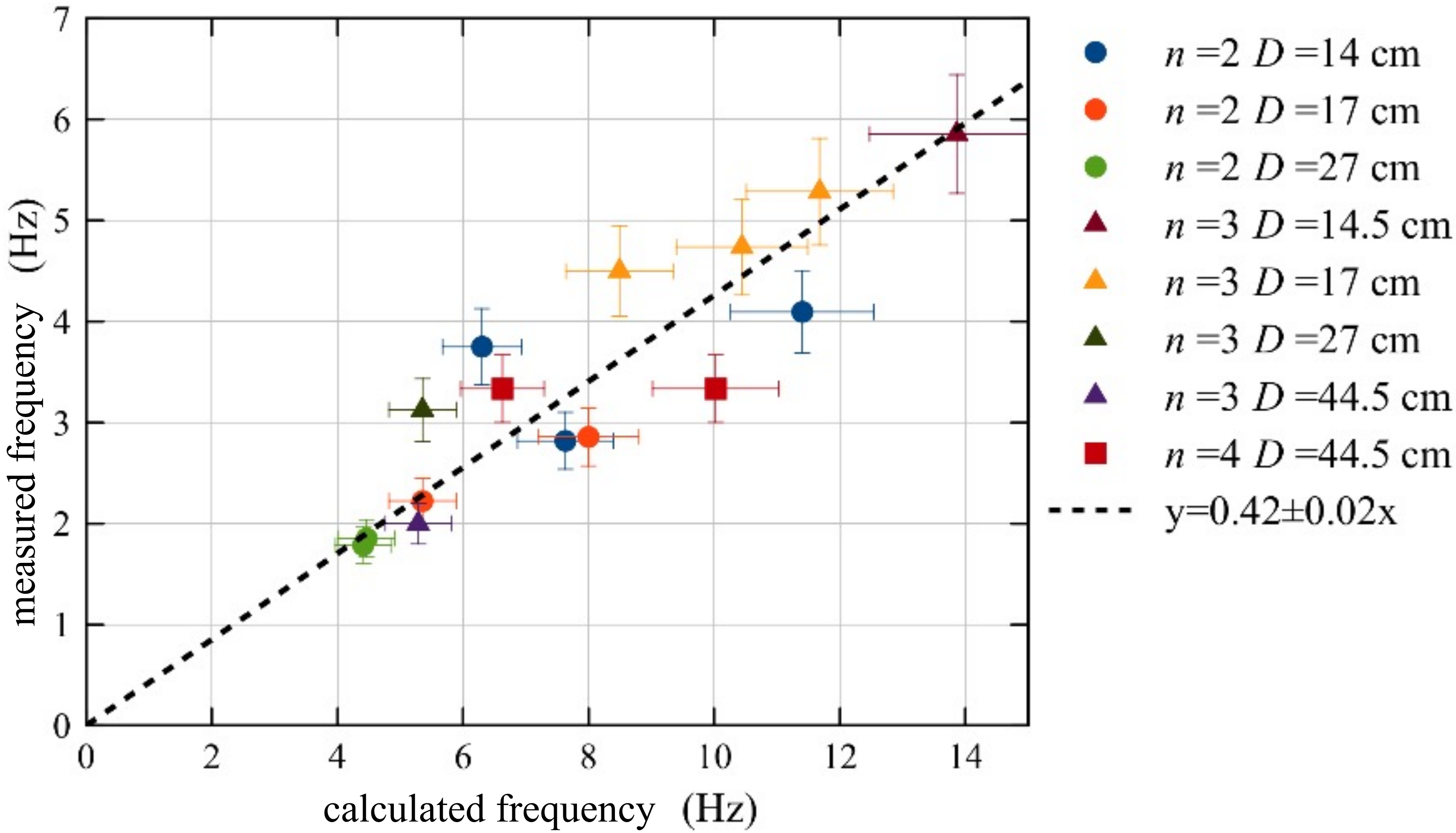
(In the co-rotating frame)

Wavelength of the polygons

(at $\bar{r}$)



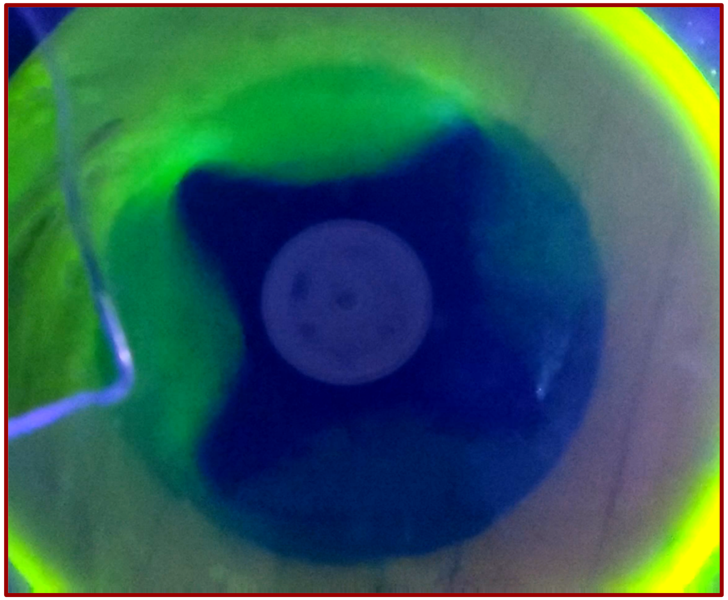
V.5. Comparison

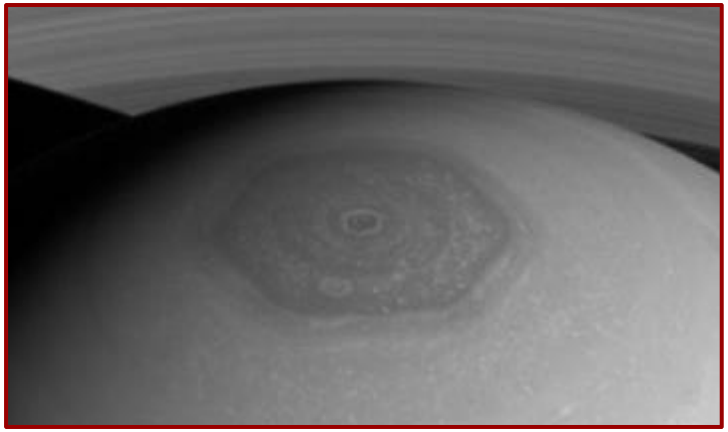


VI. Summary

VI. Summary

- Polygon vortices
  - Shear insatbility
  - Similar to surface waves
  - Modified water surface wave dispersion relation
  - $f_{\text{wave}} = \sqrt{\frac{g'}{2\pi}\frac{1}{\lambda}\tanh\left(2\pi\frac{h'}{\lambda}\right)}$





APPENDIX

References

- R. BERGMANN, L. TOPHØJ, T. A. M. HOMAN, P. HERSEN, A. ANDERSEN, & T. BOHR. 2011 Polygon formation and surface flow on a rotating fluid surface. J. Fluid Mech. 679, 415–431.
- B.BACH, E. C.LINNARTZ, M. H. VESTED, A. ANDERSEN AND T. BOHR. 2014 From Newton's bucket to rotating polygons:experiments on surface instabilities inswirling flows. J. Fluid Mech. 386-403.
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- J. M. LOPEZ, F. MARQUES, A. H. HIRSA AND R. MIRAGHAIE. 2004 Symmetry breaking in free-surface cylinder flows. J. Fluid Mech. 502, 99-126.
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Kelvin-Helmholtz-instability example

Calculation of f_{wave}

$$\left| \frac{1}{T_{\mathrm{pattern}}^{\mathrm{lab}}} - \frac{1}{T_{\mathrm{tank}}^{\mathrm{lab}}} \right| = f_{\mathrm{pattern}}^{\mathrm{co-rot}}$$

$$f_{\mathrm{pattern}}^{\mathrm{co-rot}} \cdot n = f_{\mathrm{wave}}$$

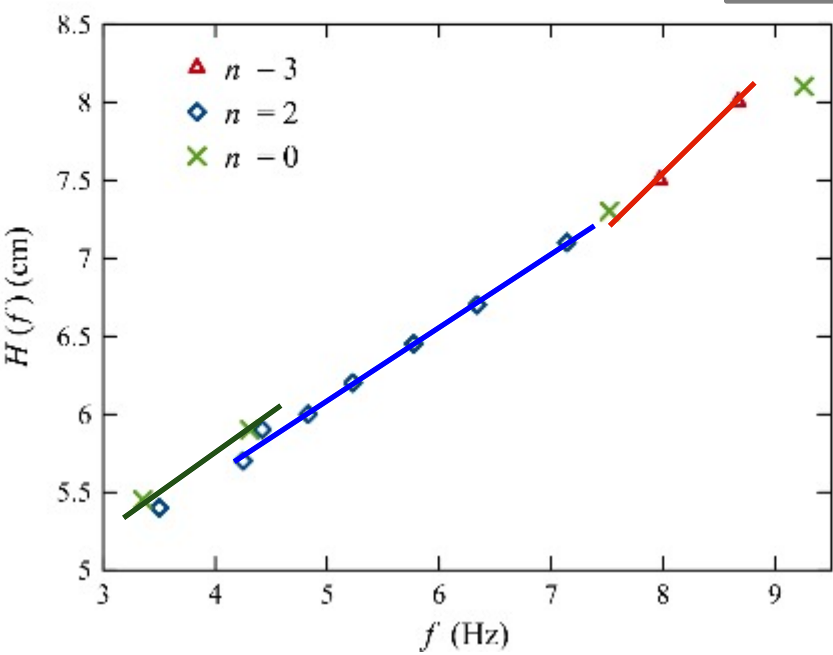
Wave number (2, 3 or 4)

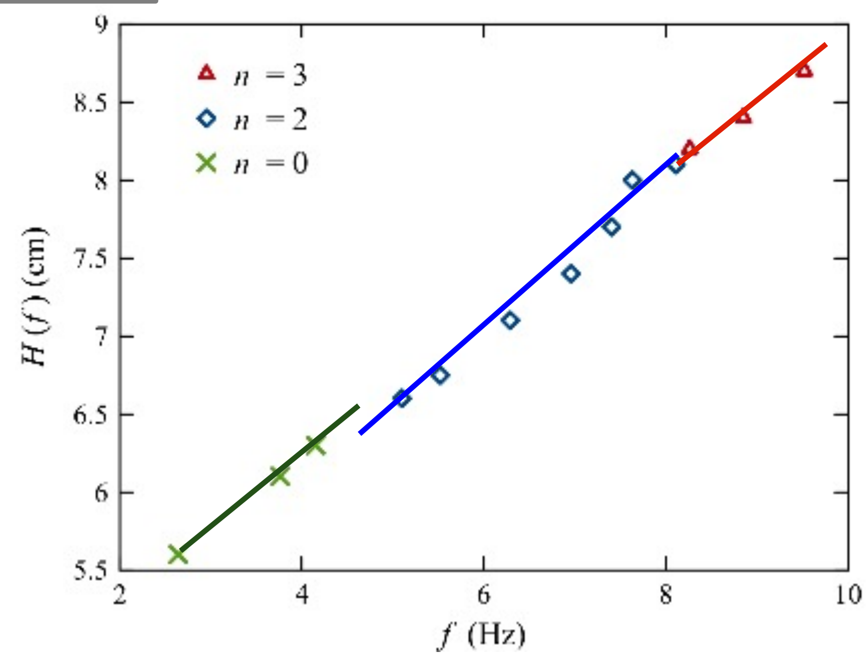
System energy investigation

$H_0 = 4.5$ cm

$R = 7.5$ cm

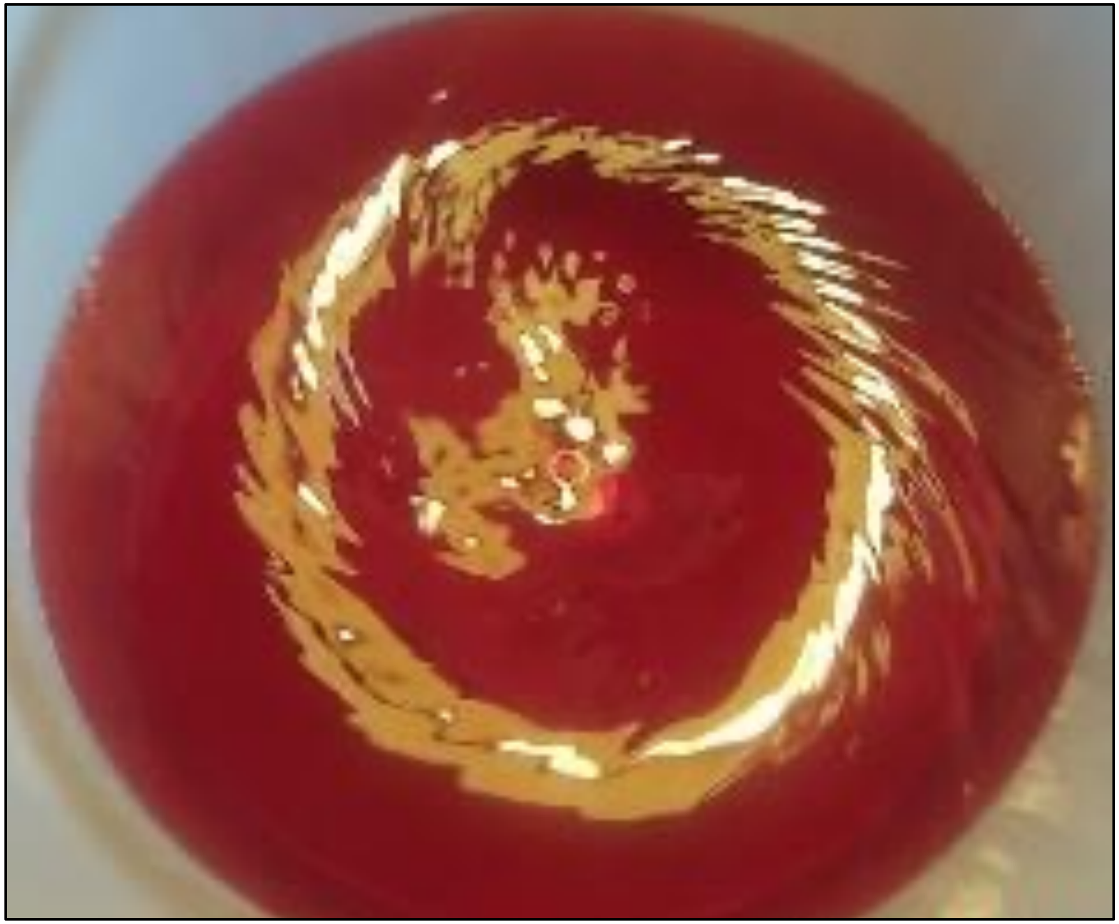
$H_0 = 5$ cm

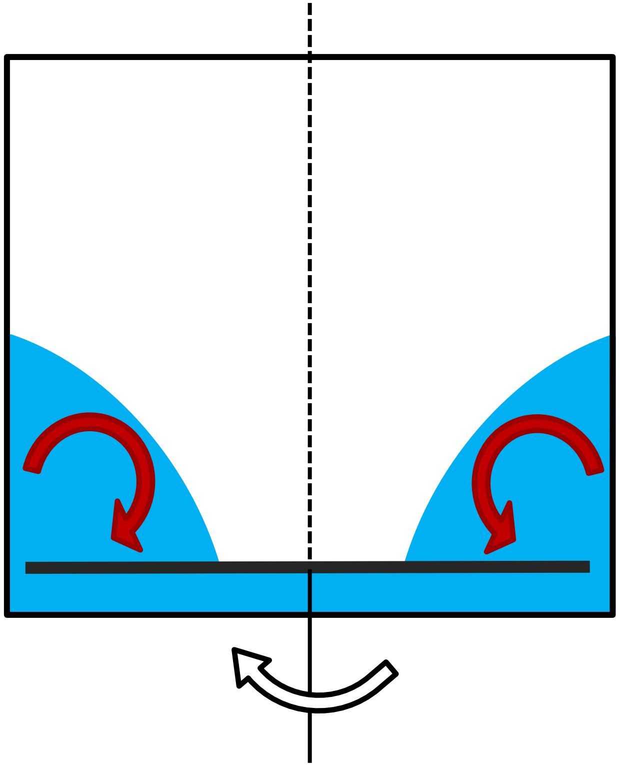




- Conclusions
  - Jump to a higher n → break in the line
  - The system is at a lower potential
  - There is a tendency, but not strong enough
  - Further investigations required

Secondary vortices





Standing waves?

- There are no stationary nodes, not even in a co-rotating frame