Introduction to photonic quantum machine learning

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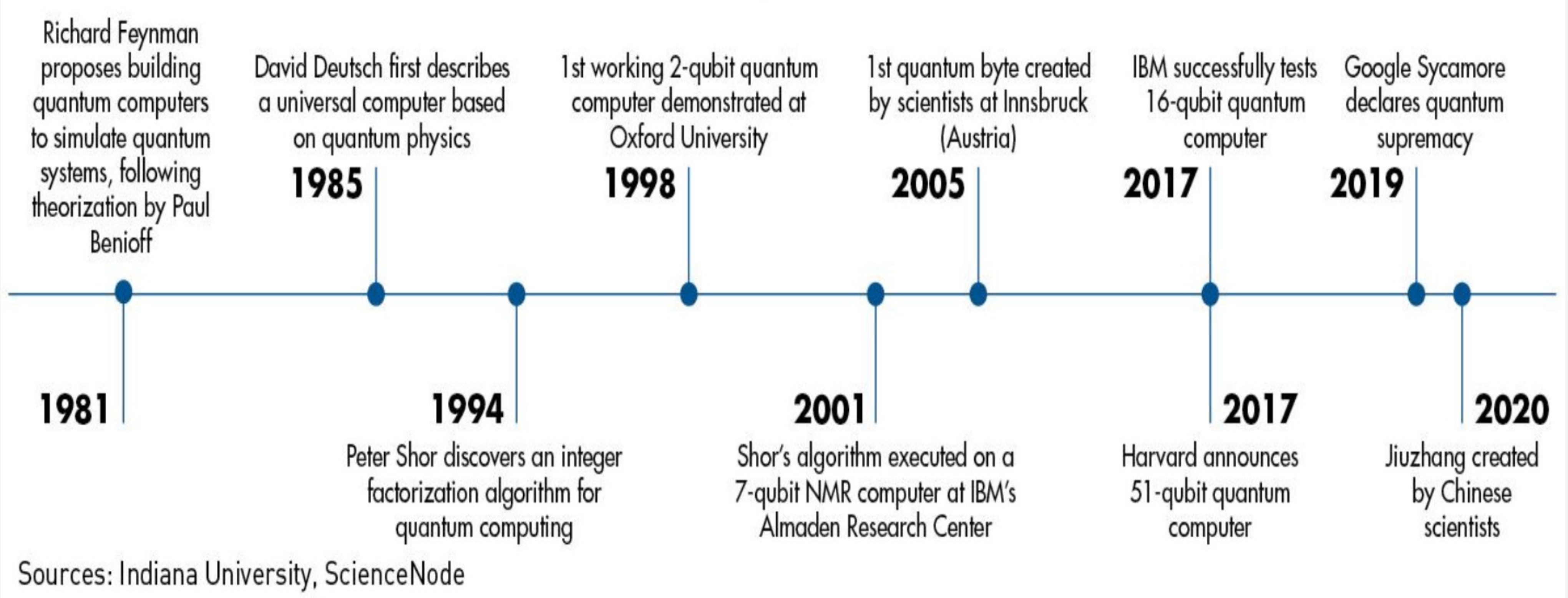
Wigner GPU Day, 2021

Agenda

- Motivation
- Basic ML Concepts
- Quantum Computing with Photons
- Photonic Quantum Machine Learning
- Numerical Experiments

Motivation



- Google q-supremacy (2019) https://www.nature.com/articles/s41586-019-1666-5
- Photonic device by USTC (Jiuzhang 2.0) (2021) https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.127.180502

Motivation

- **Today: Noisy Intermediate-scale devices (NISQ)**
  - <1000 qubits
  - Short coherence-time
  - Cross-talk
  - Gate & readout noise
  - High photon loss rate (above 50%)
  - Dark counts

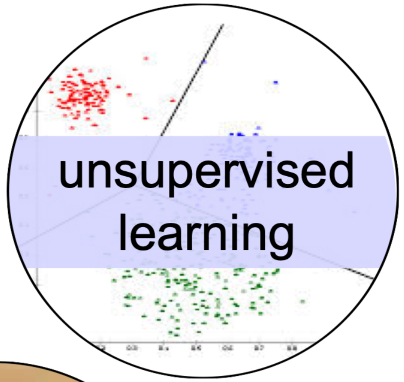
Motivation

- **NISQ-era candidates for practical quantum advantage:**
  - Simulation of quantum chemistry and many-body systems
  - Variational quantum optimization methods like QAOA
  - **Quantum Machine Learning**

Machine Learning

- Instead of implementing rules, let the computer learn the rules.

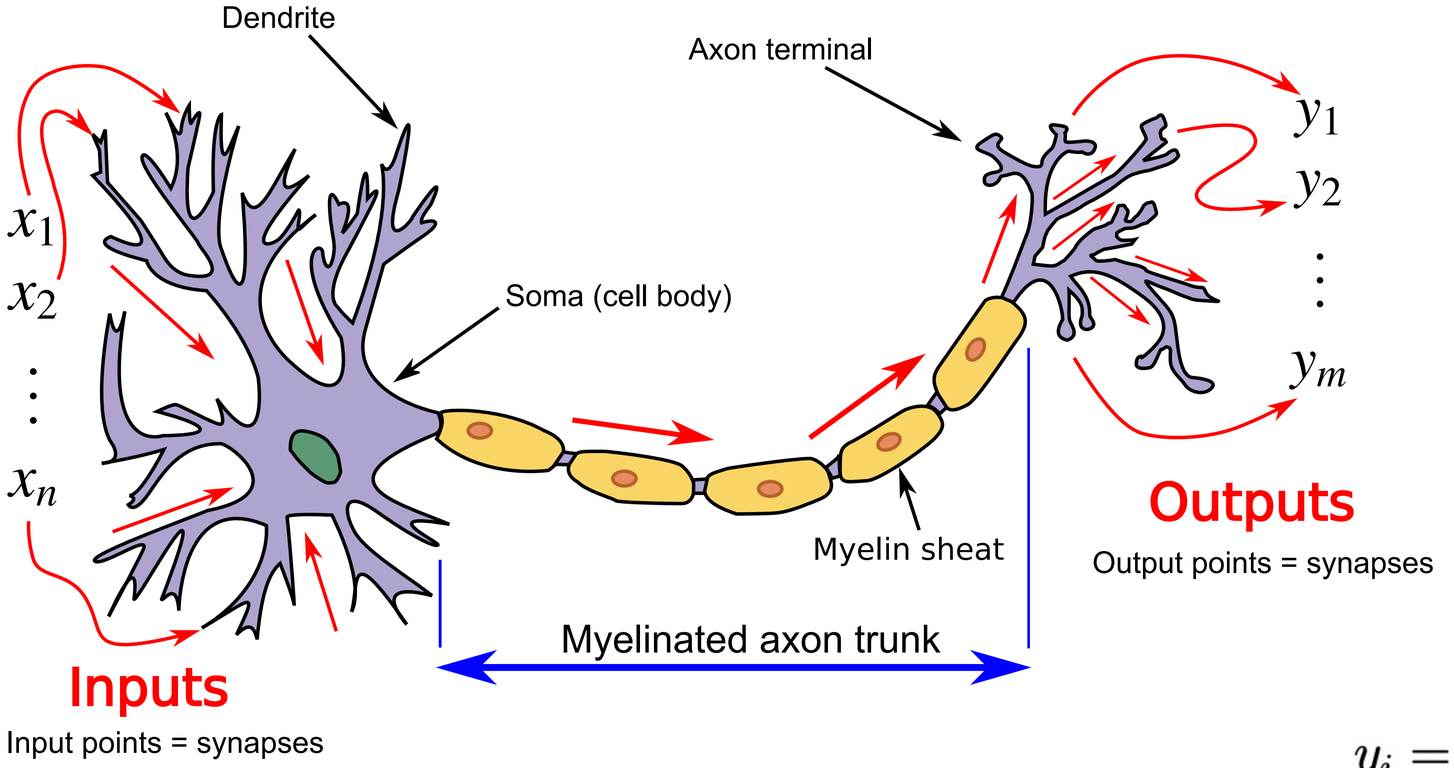
Learn patterns in
labeled data
e.g. cats vs dogs



Learn patterns in
unlabeled data
e.g. clustering

Learn to solve a control
problem or to navigate in an
environment
e.g. Atari games

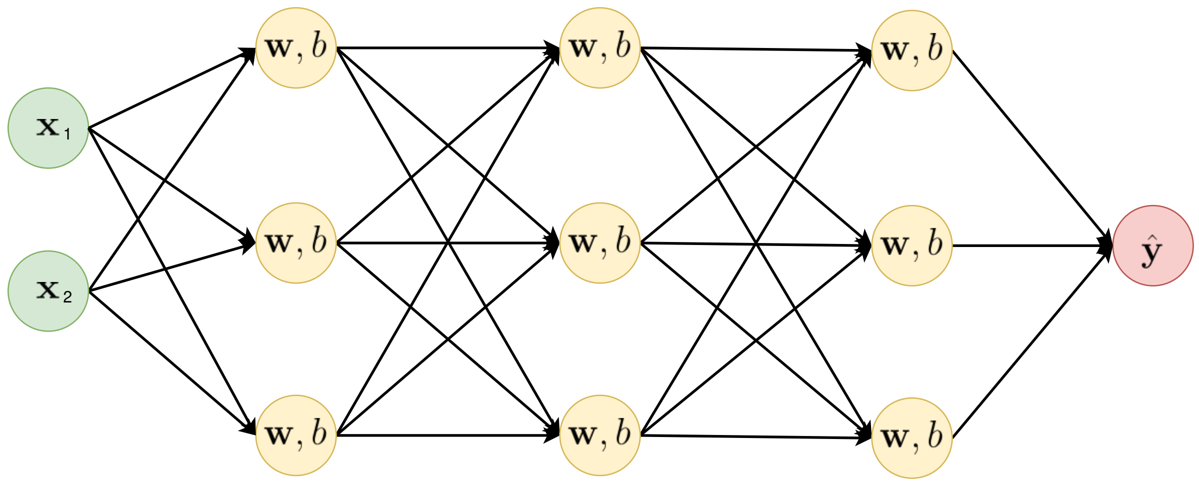
Machine Learning – Neural networks



- Our brain is good at solving problems. Idea: try to model the brain and simulate it.
- Input dendrides receive voltage spikes with frequencies x_j
- Output synapses fire with frequencies y_i
- U_0 is the threshold potential
- θ is the step funcition

$$y_i = \theta\left(\sum_j J_{ij} x_j - U_0\right)$$

Machine Learning – Neural networks

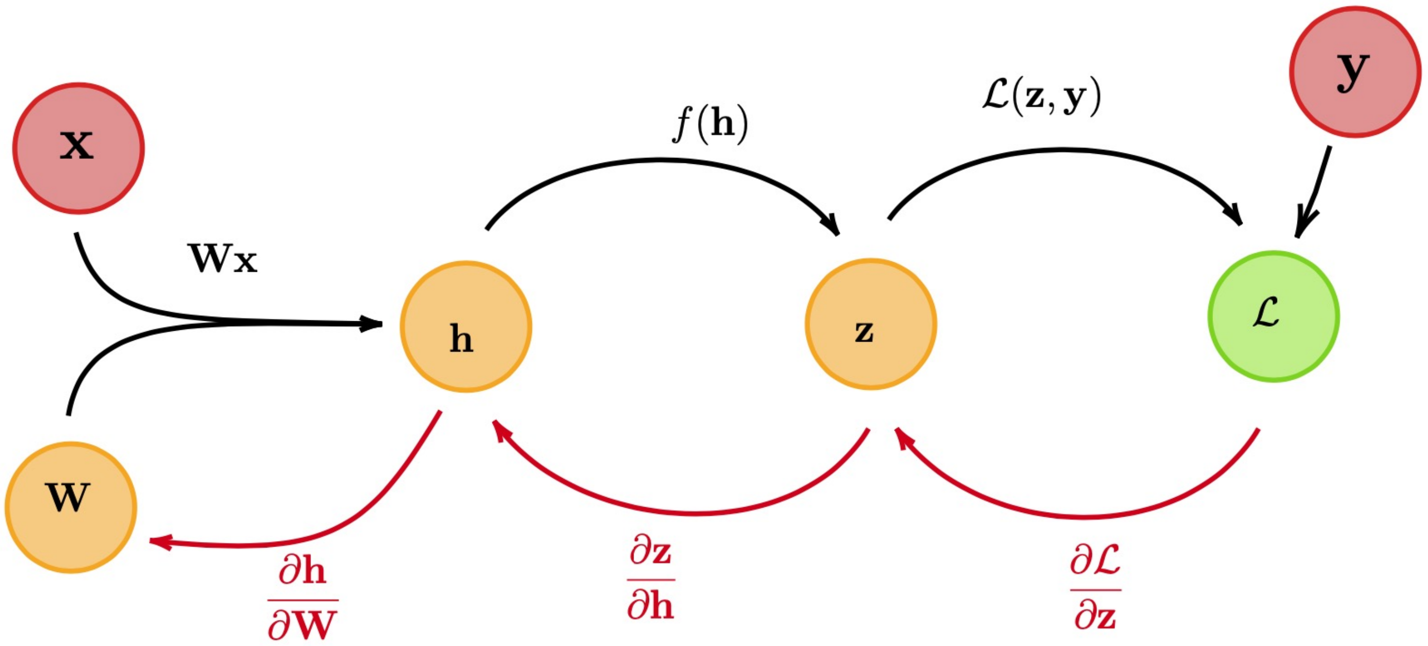


$$\mathbf{y} = g\left(\mathbf{W}^\top \mathbf{x} + \mathbf{b}\right)$$

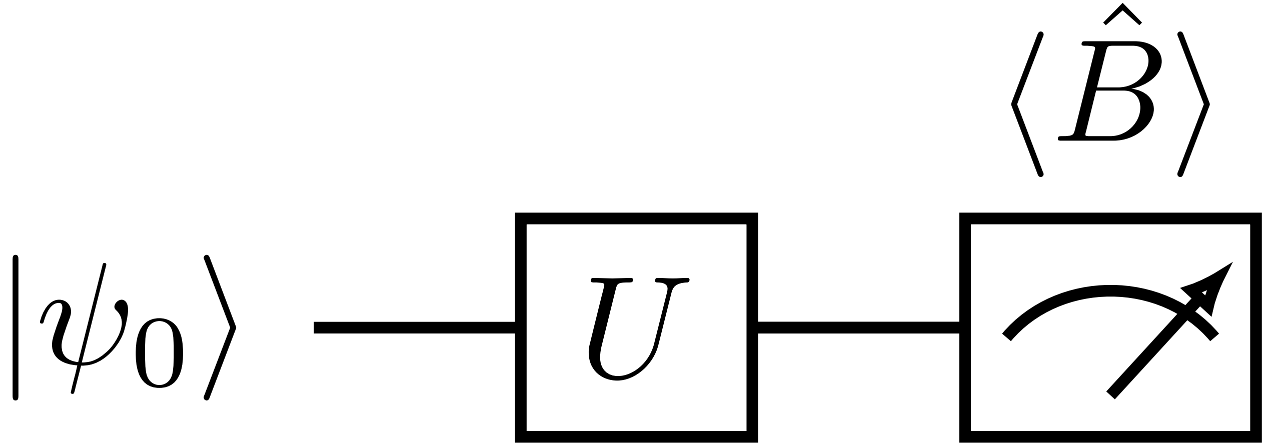
- Goal: find W and b that minimize the error.
- Neural Networks are universal function approximators

How to train Neural Networks?

- Define a loss function: $L = \frac{1}{N}\sum_{k=1}^{N} \mathcal{L}(\mathbf{y}_k, f(\mathbf{x}_k; \boldsymbol{\theta}^{(t)}))$
- Update parameters: $\theta_j^{(t+1)} \leftarrow \theta_j^{(t)} - \alpha_j^{(t)} \left. \frac{\partial L}{\partial \theta_j} \right|_{\theta_j = \theta_j^{(t)}}$



Quantum Computation – Circuit Model



Photonic Quantum Computation

Qubit paradigm

- Qubit basis: $\{|0\rangle, |1\rangle\}$
- $\dim(H) = 2^N$
- Can be realized by photons
- Common errors:
  - Bit-flip error
  - Phase-shift error
  - Gate errors
  - Measurement errors
- Cryogenic Temperature

Qumode paradigm

- Fock basis: $\{|0\rangle, |1\rangle, \ldots\}$
- $\dim(H) = \infty$
- $\dim(H) = D^N$
- Best for describing photons
- Common errors:
  - Photon loss
  - Gate errors
  - Measurement errors
- Room Temperature

Photonic Quantum Computation

Equivalent descriptions

Fock-space description ⟷ **Phase-space description**

$$\hat{a}_k = \frac{\hat{X}_k + i\hat{P}_k}{\sqrt{2}}$$

$$\hat{a}_k^\dagger = \frac{\hat{X}_k - i\hat{P}_k}{\sqrt{2}}$$

Quadrature operators →

$$\hat{X}_k = \frac{\hat{a}_k + \hat{a}_k^\dagger}{\sqrt{2}}$$

$$\hat{P}_k = \frac{\hat{a}_k - \hat{a}_k^\dagger}{i\sqrt{2}}$$

Defines an infinite dimensional Hilbert space

$$|n_1, \ldots, n_k, \ldots\rangle$$

Continuous basis →

$2N$-dimensional symplectic phase-space

$$|\mathbf{x}\rangle \qquad |\xi\rangle$$

Linear operators can have non-linear counterparts →

Phase-space description

- We work in the quadrature basis: $$\hat{X}_j = X_j \left|\mathbf{x}\right\rangle$$

- We can calculate the Wigner-function:

$$W_\rho(\mathbf{x}, \mathbf{p}) = \frac{1}{\pi^N} \int_{\mathbb{R}^N} \left\langle \mathbf{x} + \mathbf{y} \right| \rho \left| \mathbf{x} - \mathbf{y} \right\rangle e^{-2i\mathbf{p}\mathbf{y}} \mathrm{d}^N \mathbf{y}$$

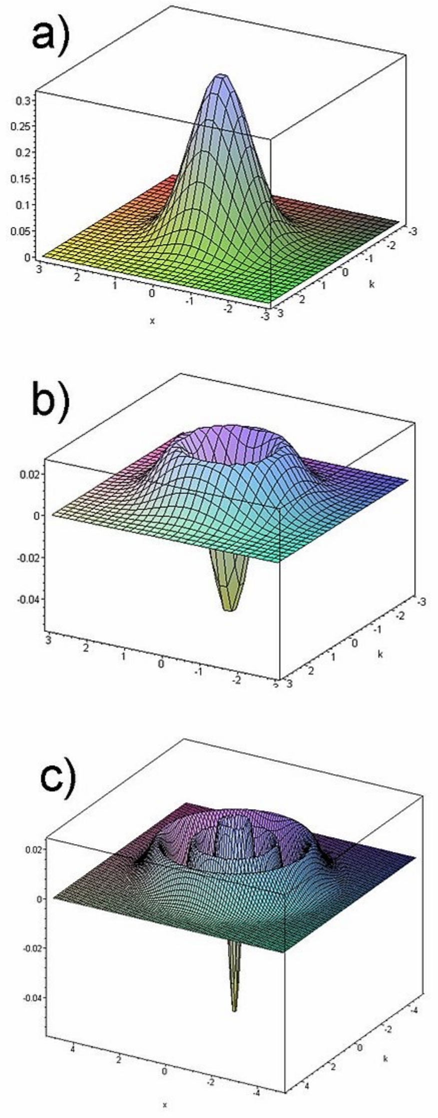
Phase-space description

- The Wigner function can be used to calculate expectation values:

$$\langle X_1 \rangle = \int_{\mathbb{R}^{2N-1}} W_\rho(\mathbf{x}, \mathbf{p}) \mathrm{d}p_1 \cdots \mathrm{d}p_N \mathrm{d}x_2 \cdots \mathrm{d}x_N$$

- And also the purity of the state:

$$\mu_\rho = \mathrm{Tr}\left[\rho^2\right] = (2\pi)^N \int_{\mathbb{R}^{2N}} \left(W_\rho(\mathbf{x}, \mathbf{p})\right)^2 \mathrm{d}^N\mathbf{x}\mathrm{d}^N\mathbf{p}$$



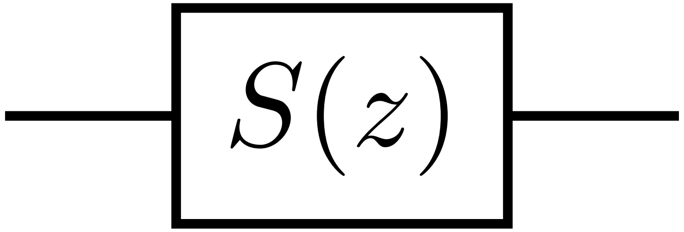
Source: Wikipedia

Gaussian states

- Quadrature vector: $$\mathbf{R}^\top = (\hat{X}_1, \hat{P}_1, \hat{X}_2, \hat{P}_2, ..., \hat{X}_k, \hat{P}_k, ..., \hat{X}_M, \hat{P}_M)$$
- First moment: $$d_j = \langle \hat{R}_j \rangle_\rho$$
- Second moment: $$\sigma_{ij} = \langle \hat{R}_i \hat{R}_j + \hat{R}_j \hat{R}_i \rangle_\rho - 2\langle \hat{R}_i \rangle_\rho \langle \hat{R}_j \rangle_\rho$$
- Gaussian state: $$W_\rho(\boldsymbol{\xi}) = \frac{1}{\pi^N \sqrt{\det \boldsymbol{\sigma}}} e^{-(\boldsymbol{\xi}-\mathbf{d})^\top \boldsymbol{\sigma}^{-1} (\boldsymbol{\xi}-\mathbf{d})}$$
- Vacuum state: $$W_{|0\rangle}(X, P) = \frac{1}{\pi} e^{-X^2 - P^2}$$

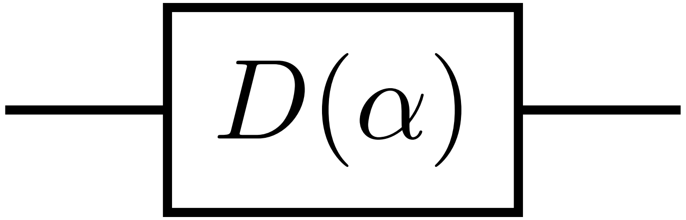
Universal Photonic Quantum Gate Set

Squeezing



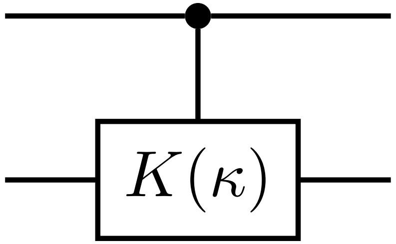
$$\hat{S}(z) = \exp\left[\frac{1}{2}\left(z^*\hat{a}^2 - z\hat{a}^{\dagger 2}\right)\right]$$

Displacement



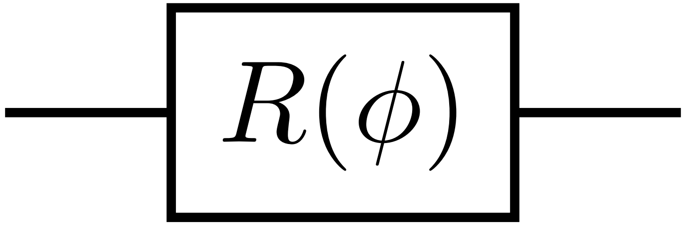
$$\hat{D}(\alpha) = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}$$

Cross-Kerr



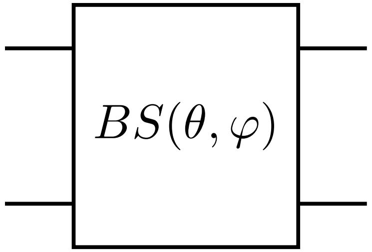
$$\hat{CK}(\kappa) = e^{i\kappa\hat{n}_j\hat{n}_k}$$

Rotation



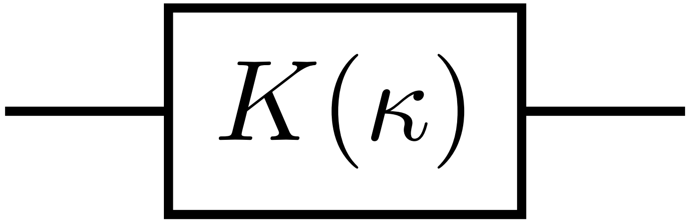
$$\hat{R}(\phi) = e^{i\phi\hat{a}^\dagger\hat{a}}$$

Beam-splitter



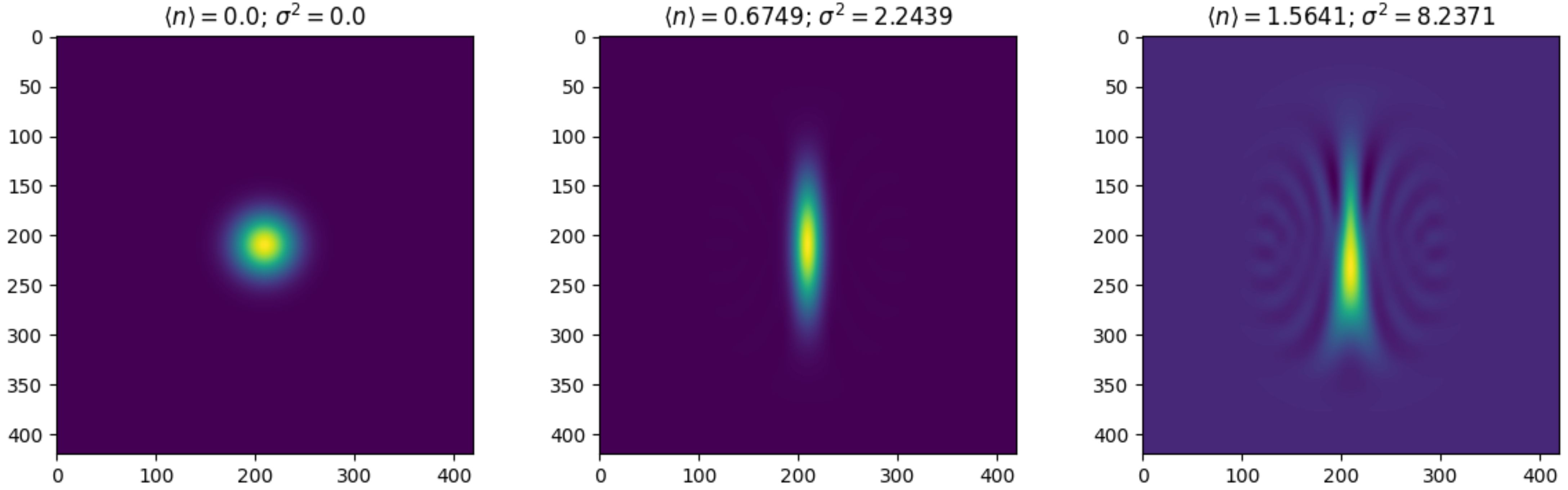
$$\hat{BS}(\theta,\varphi) = \exp\left[\theta\left(e^{i\varphi\hat{a}_j\hat{a}_k^\dagger} - e^{-i\varphi\hat{a}_j^\dagger\hat{a}_k}\right)\right]$$

Kerr



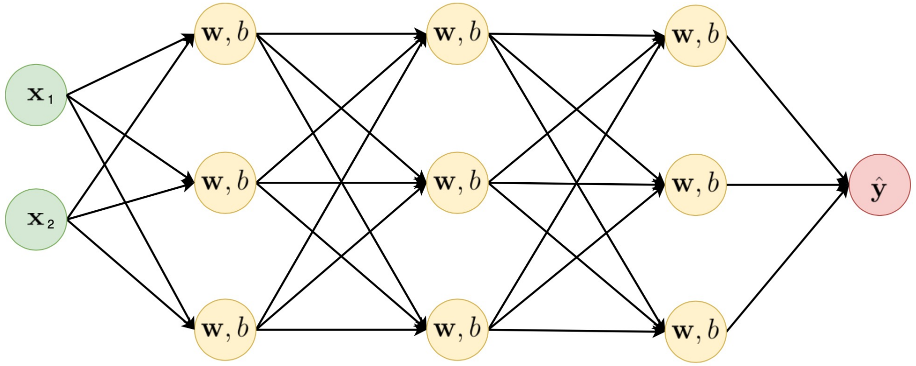
$$\hat{K}(\kappa) = e^{i\kappa\hat{n}^2}$$

Gaussian & Non-Gaussian States



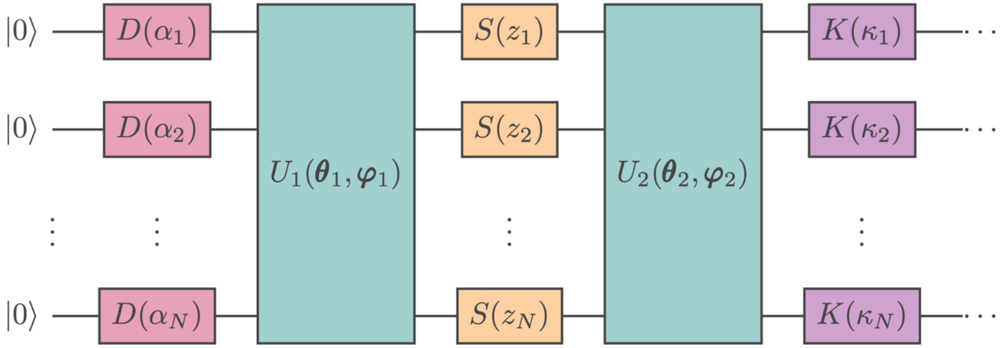
Cubic phase: $V(\gamma) = e^{i\gamma x^3/(3\hbar)}$

Continuous-Variable QNNs



$$\mathcal{L}(\mathbf{x}) = g(\mathbf{W}^\top \mathbf{x} + \mathbf{b})$$

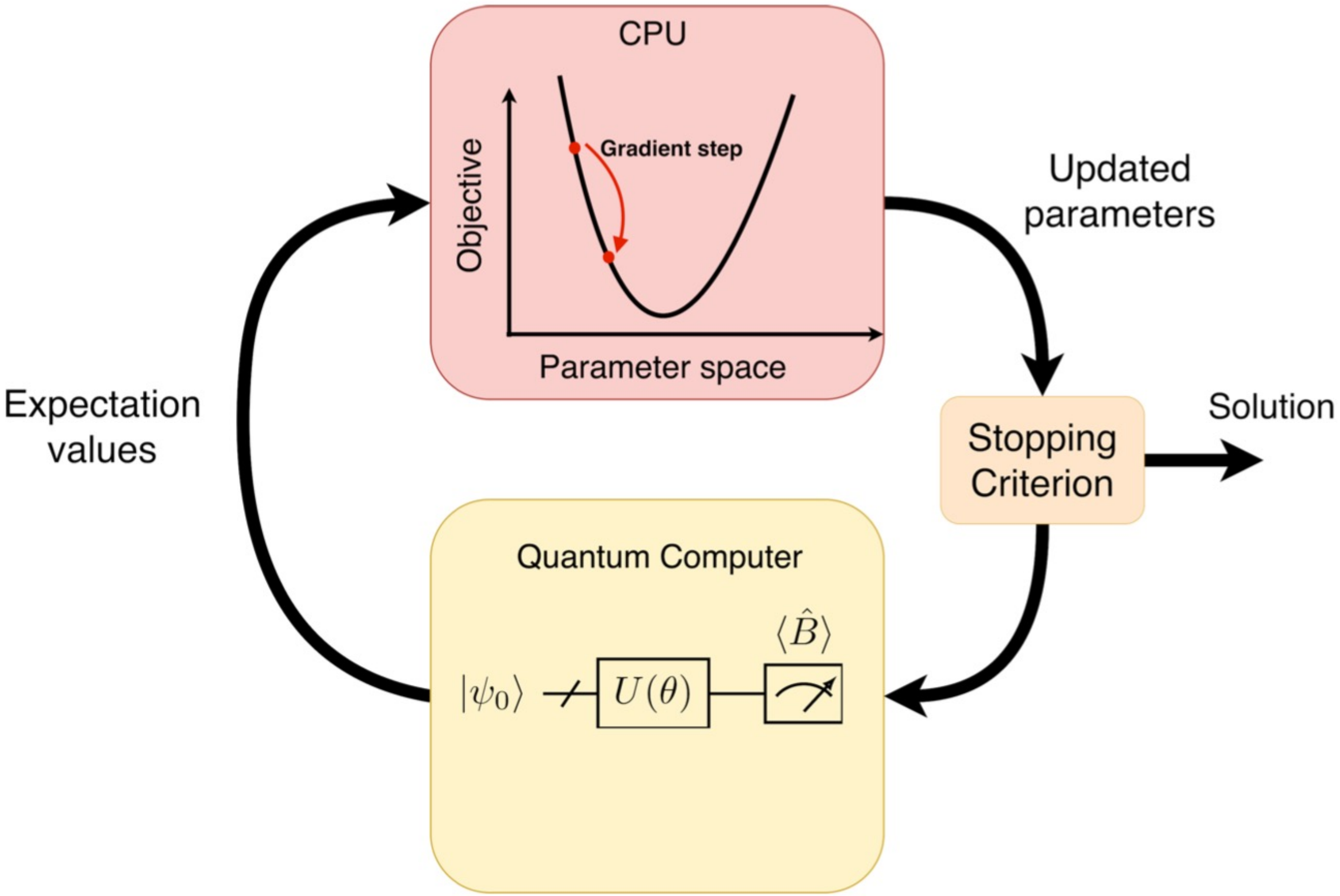
Universal function approximator



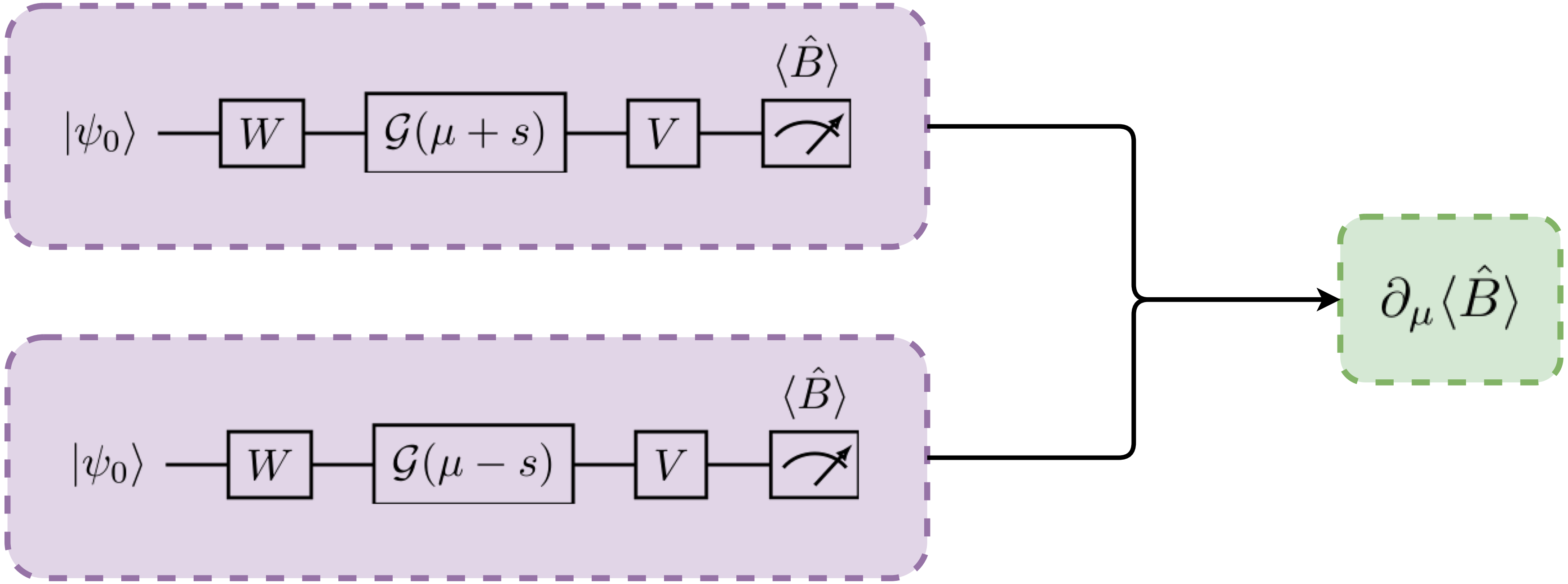
$$\mathcal{L}(\boldsymbol{\xi}) = K(\mathbf{M}\boldsymbol{\xi} + \boldsymbol{\alpha})$$

Universal unitary approximator

Hybrid Training Loop



Training Quantum Neural Networks



Evaluating analytic gradients on quantum hardware

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(Received 9 January 2019; published 21 March 2019)

Numerical experiments

- Classification
- Regression
- VQE
- Reinforcement Learning

Strawberry Fields:
A Software Platform for Photonic Quantum Computing

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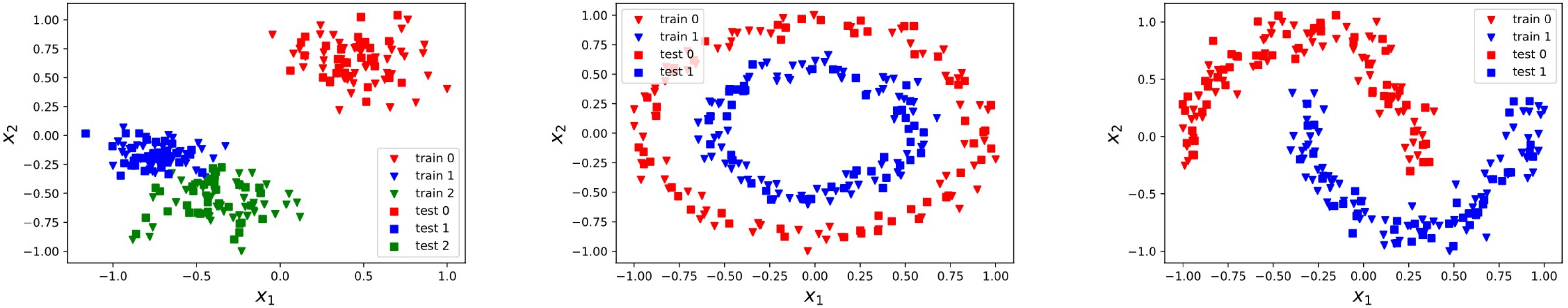
Xanadu, 372 Richmond St W, Toronto, M5V 1X6, Canada

TensorFlow

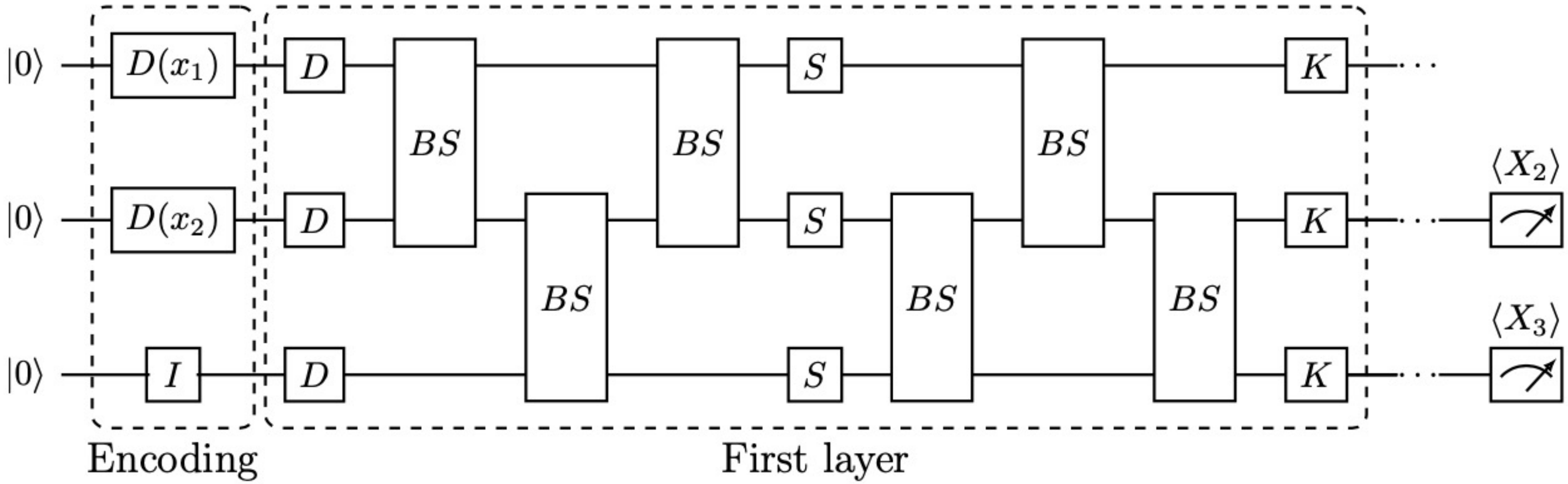
Classification



Encoding spacial data: $$(x_1, x_2) \mapsto \hat{D}_1(x_1) \otimes \hat{D}_2(x_2) \otimes I_3 \left|0, 0, 0\right\rangle$$

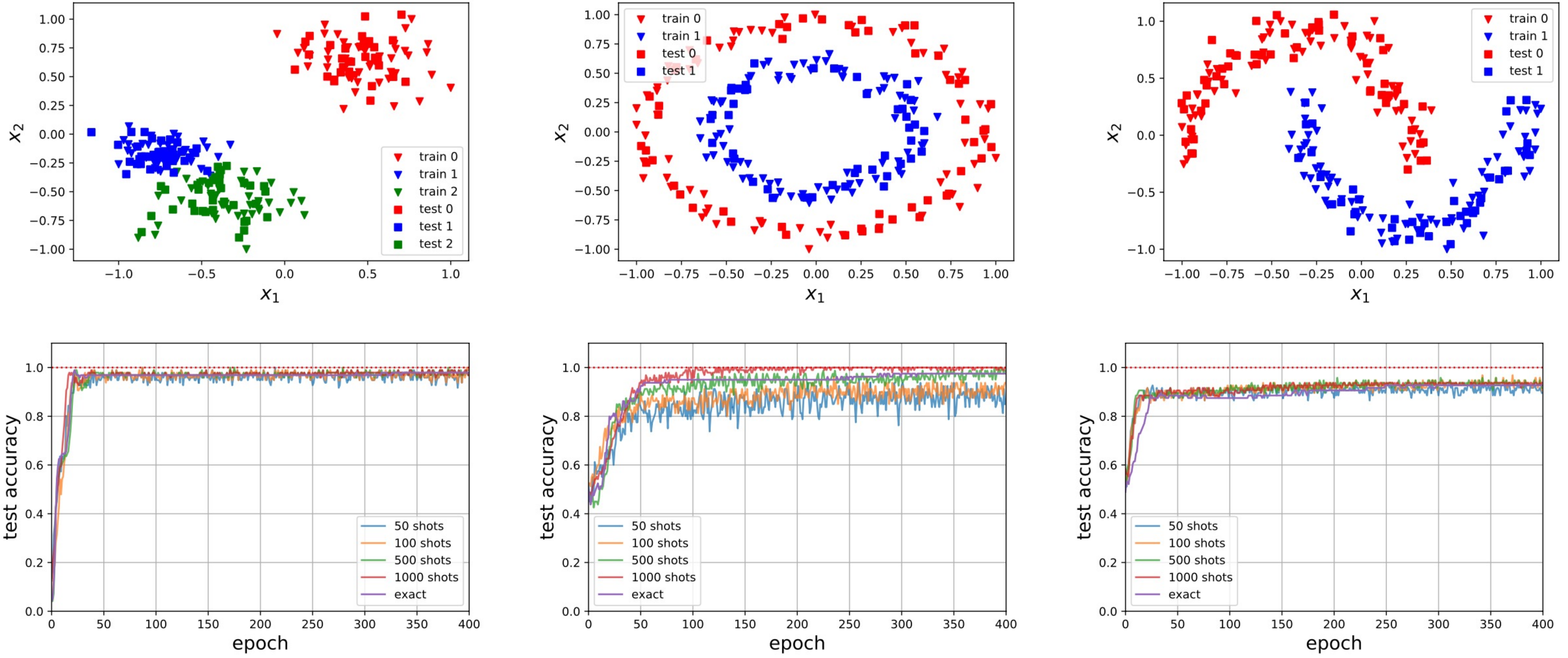
Predicted categories: $$\mathbf{y}^{\text{pred}} = \texttt{Softmax}\begin{bmatrix} \text{abs}\left(\langle \hat{X}_2 \rangle_{\rho,k}\right) \\ \text{abs}\left(\langle \hat{X}_3 \rangle_{\rho,k}\right) \end{bmatrix}$$

Classification

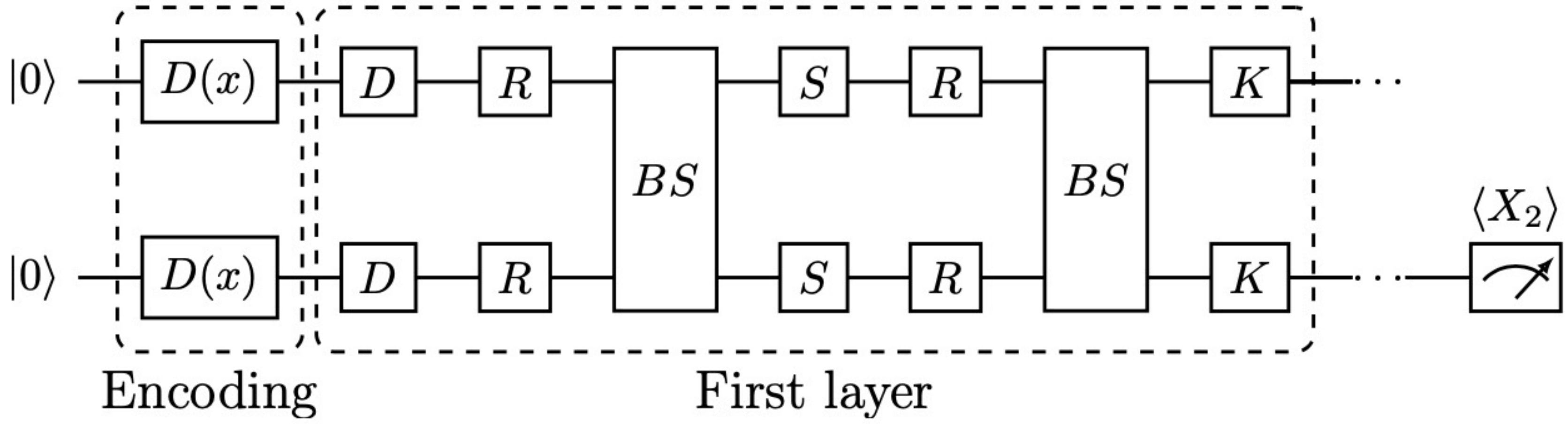


$$\mathcal{L} = \texttt{CrossEntropy}(\mathbf{y}^{\mathrm{pred}}, \mathbf{y}^{\mathrm{true}}) + \frac{\lambda}{|\mathcal{B}|} \sum ||W^{\mathrm{active}}||^2$$

Classification Results



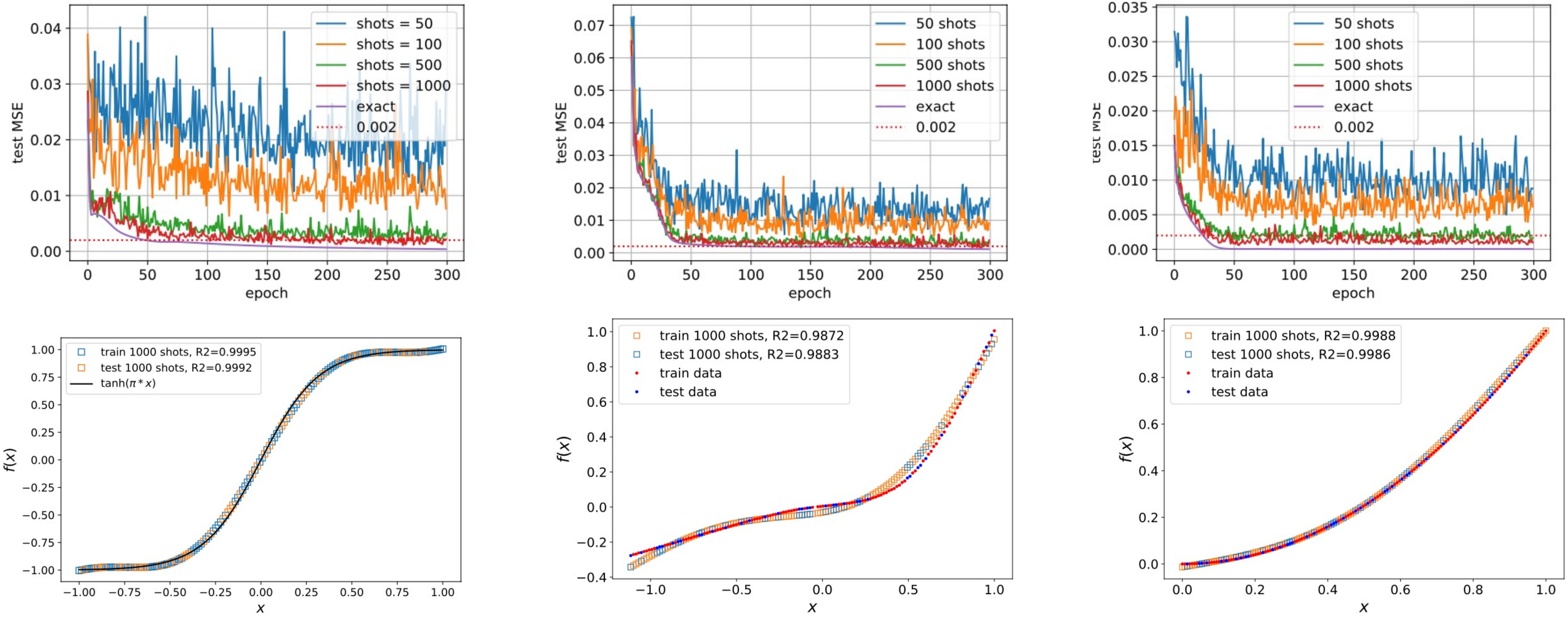
Regression



$$x \mapsto \hat{D}_1(x) \otimes \hat{D}_2(x)\,|0,0\rangle$$

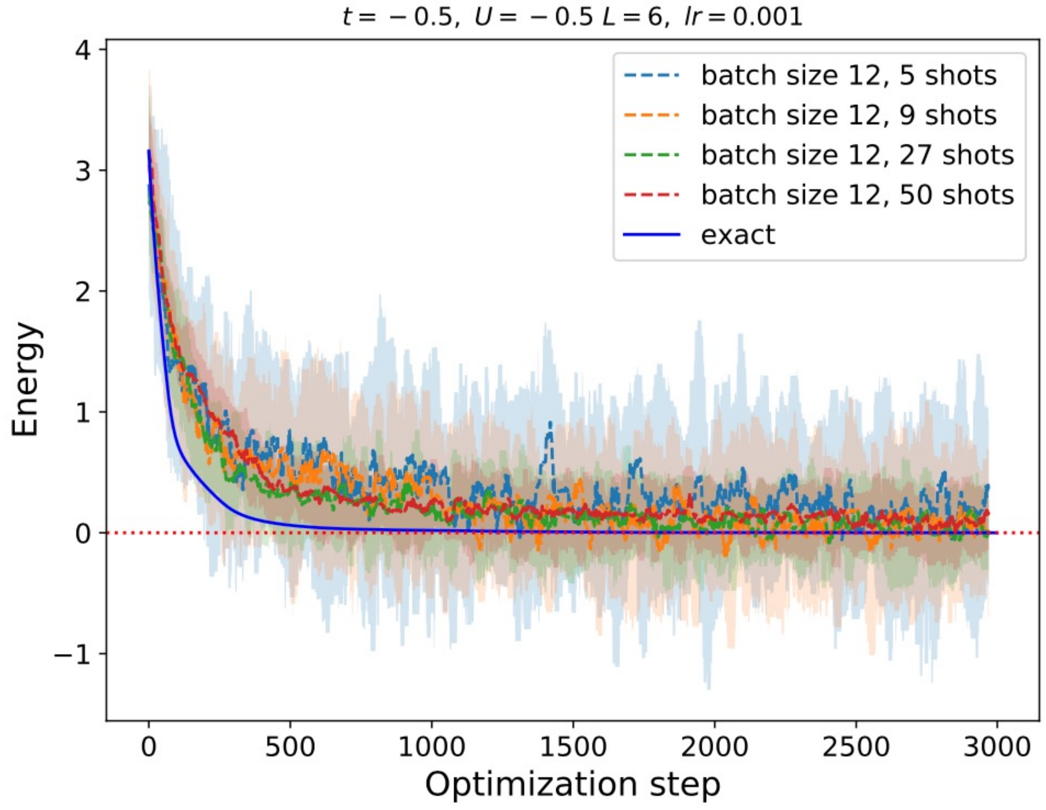
$$\mathcal{L} = \frac{1}{|\mathcal{B}|} \sum_{(x,y)\in\mathcal{B}} \left(y - \langle \hat{X}_2 \rangle_{\rho,k}\right)^2 + \frac{\lambda}{|\mathcal{B}|} \sum ||W^{\text{active}}||^2$$

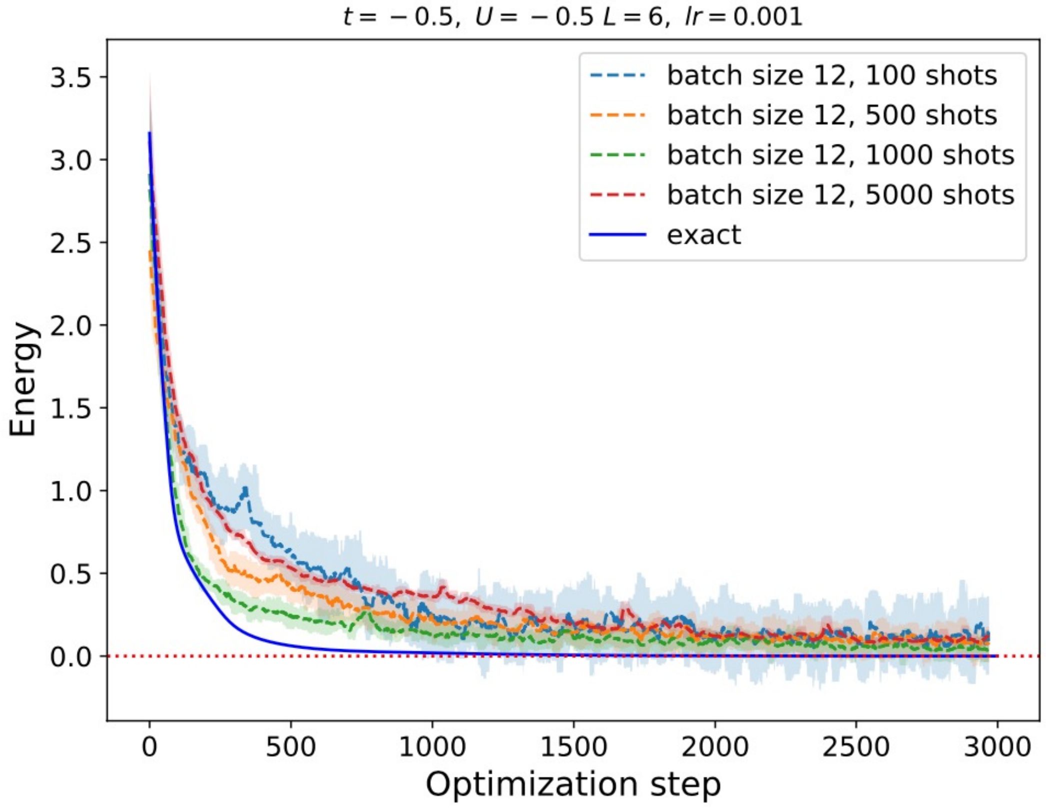
Regression Results



Bose-Hubbard model Results (VQE)

$$\hat{H} = -t \sum_{i=1}^{M-1} (\hat{a}_i^\dagger \hat{a}_{i+1} + \hat{a}_{i+1}^\dagger \hat{a}_i) + \frac{U}{2} \sum_{i=1}^{M} \hat{n}_i^2 \qquad \mathcal{L}(\boldsymbol{\theta}) = \langle \hat{H} \rangle_\rho$$





Conclusion

- We presented CV-quantum computing
- We showed the CV-QNN architecture which is analogous to the classical fully connected NNs
- We presented numerical experiments in which QNNs were used on basic datasets

Outlook

- Testing QNNs on more complex data
- Improving the QNN architecture
- Quantum-specific optimization algorithms

Thank you!

References

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