Numerical modelling of compact stars in search of extra dimensions

Anna Horváth
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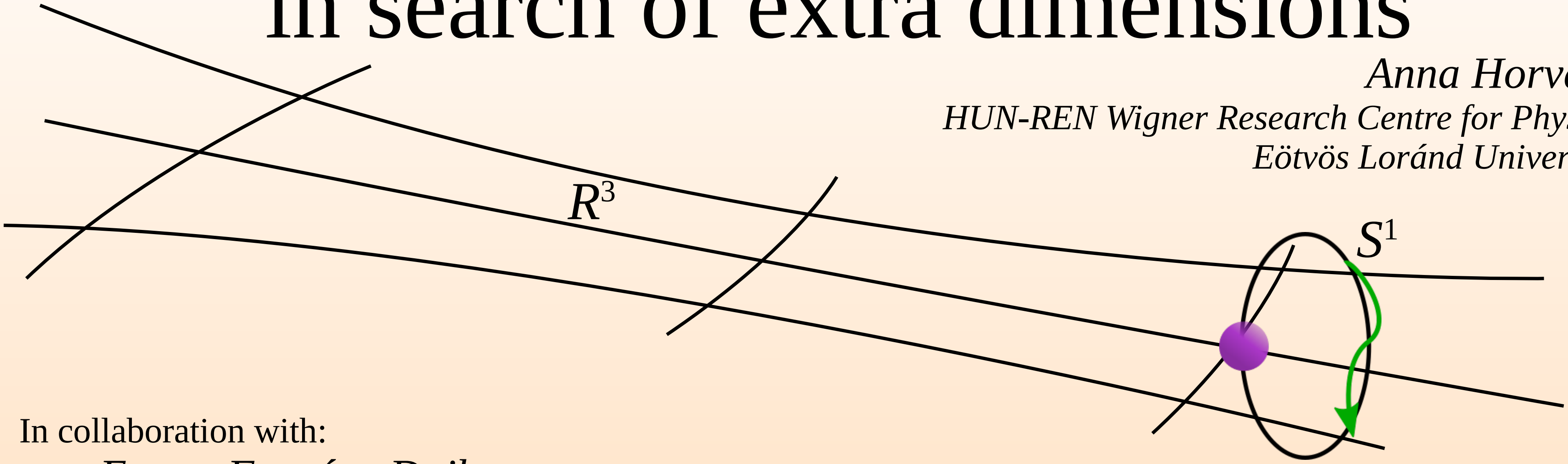

In collaboration with:
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NKFIH OTKA K147131
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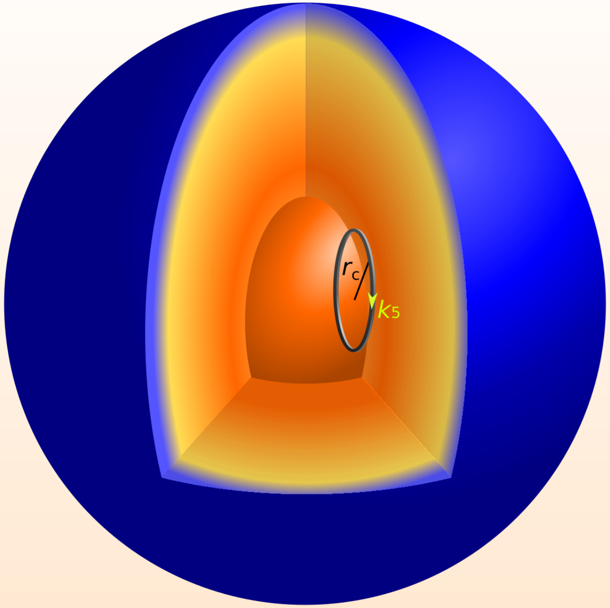
KMP-2023/101, KMP-2024/31

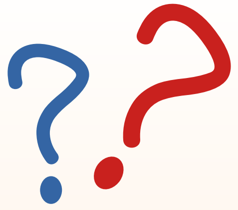
Outline of the talk

- Motivation
- Modelling stars
- Extra dimensions
- Multiple extra dimensions
- Fun fact
- Results



[1] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "Application of Kaluza-Klein Theory in Modeling Compact Stars: Exploring Extra Dimensions", MNRAS (2024)
[2] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "The effect of multiple extra dimensions on the maximal mass of compact stars in Kaluza-Klein space-time", Accepted by IJMPA (2025)
[3] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: “Speed of sound in Kaluza-Klein Fermi gas”, Sent to APP-B (2025)

Motivation



Questions of **fundamental physics**

- GR + QM = ?
- Hierarchy problem
- Dark matter, dark energy?

↓

Could be **solved** by **extra dimensions**

→ **How** do we **test** if they exist?

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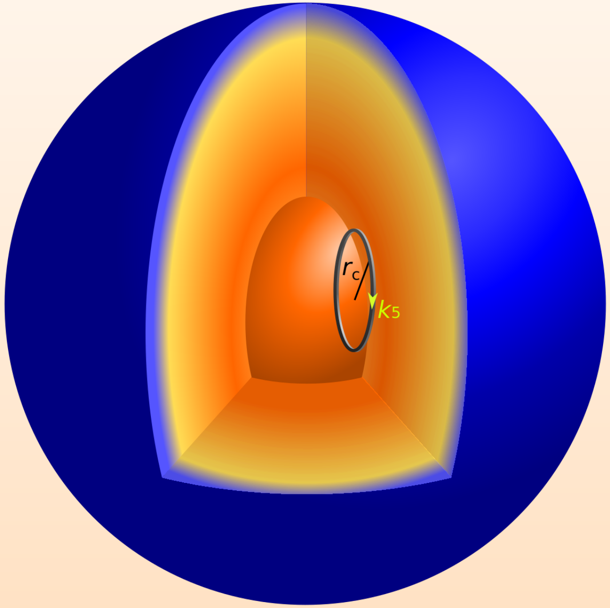

With **neutron stars**.

Very small.

Very heavy.

Very **dense**.

Also **cold**. (Hot actually, don't go there...)

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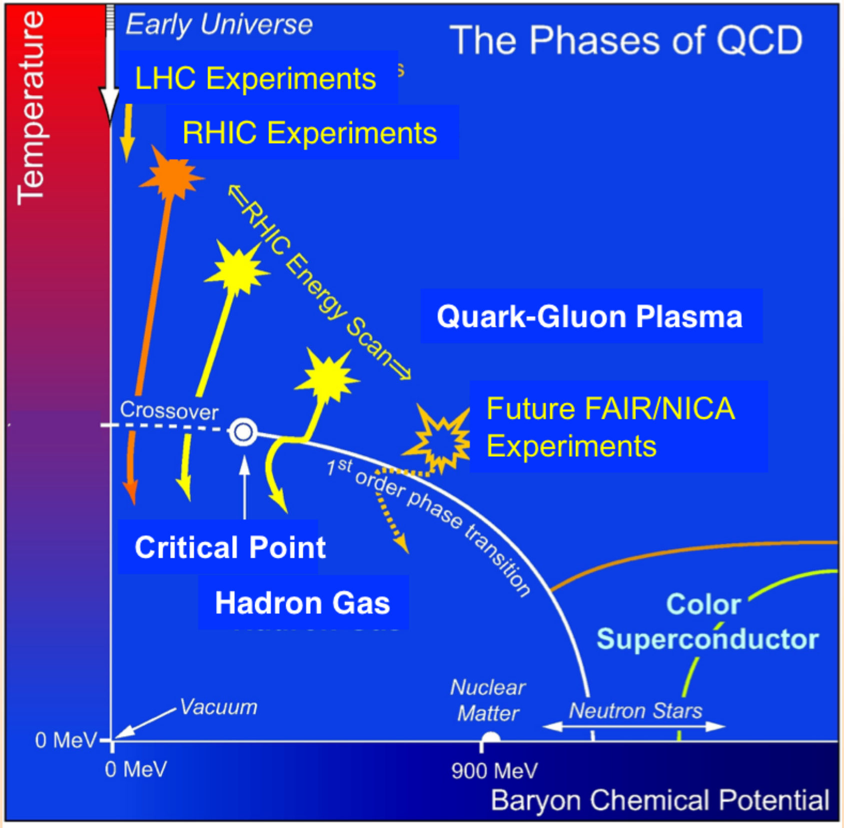
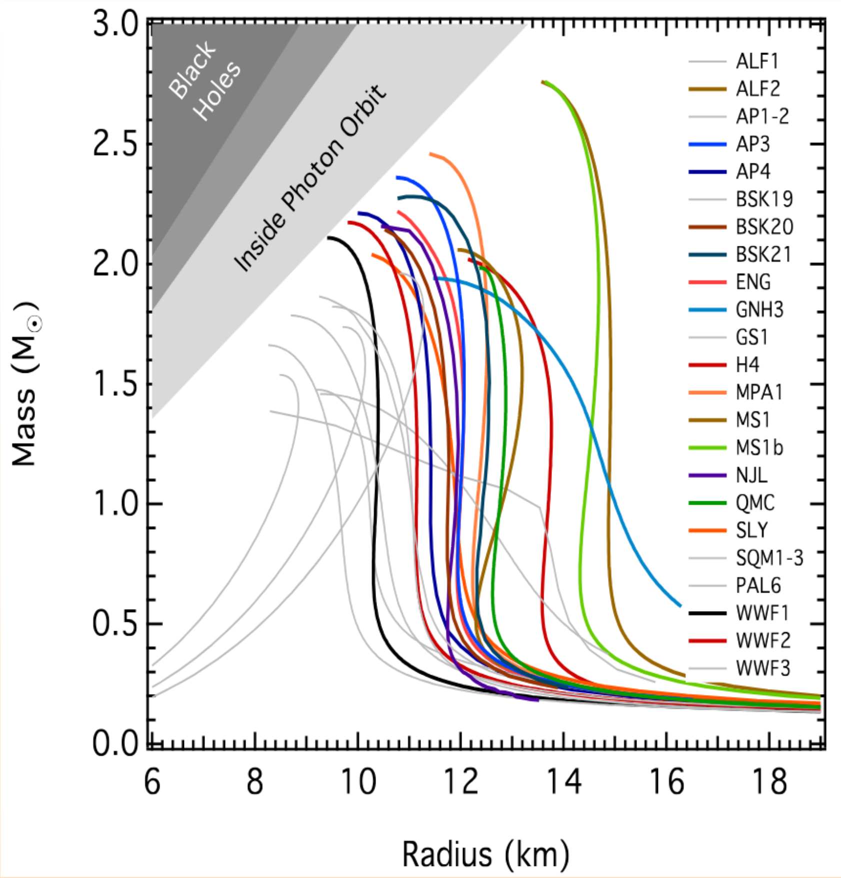


Building stars

Some measurable quantities:

- **Mass**
- Radius
- Tidal deformability (not considered)



F. Özel and P. Freire, "Masses, Radii, and the Equation of State of Neutron Stars," Ann. Rev. Astron. Astrophys. **54** (2016), 401-440 doi:10.1146/annurev-astro-081915-023322 [arXiv:1603.02698 [astro-ph.HE]].

Building stars

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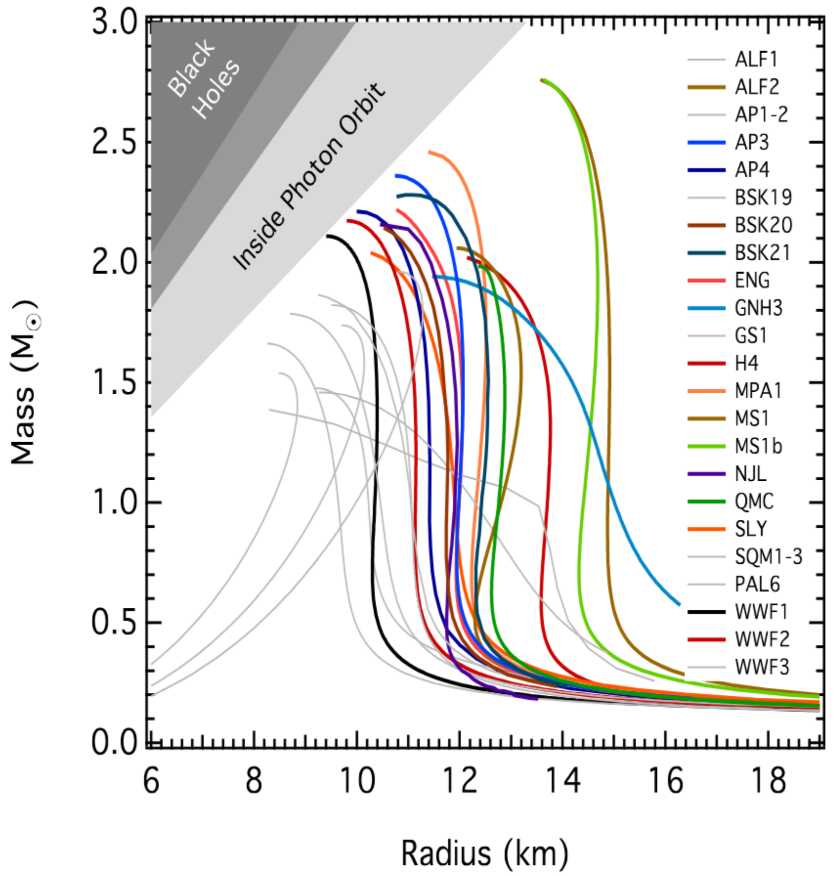
Hydrostatic equilibrium

$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{GM(r)\varepsilon(r)}{r^2}$$



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Hydrostatic equilibrium

In **general relativity**:

- the **Tolman–Oppenheimer–Volkoff** (TOV) equation

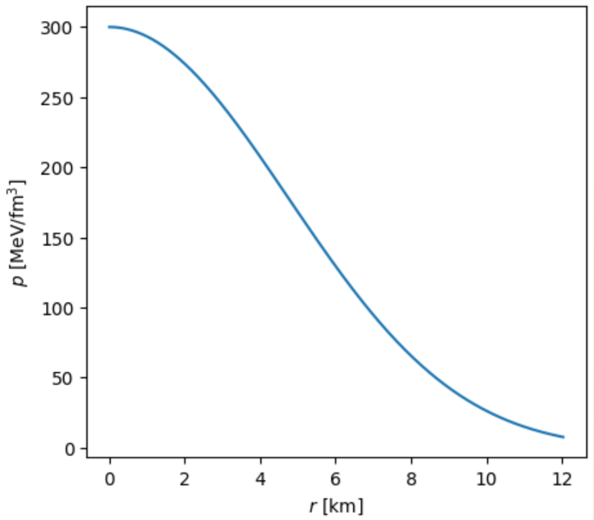
$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{GM(r)\varepsilon(r)}{r^2} \times \times \left[1+\frac{p(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{M(r)}\right]\left[1-\frac{GM(r)}{r}\right]^{-1}$$

$$M(r) = \int_0^r \mathrm{d}r' 4\pi r'^2 \varepsilon(r')$$

  Boundary conditions: central energy density ε_c, surface pressure $p(R)$

- and the **equation of state** (EoS)
  - Connects pressure (p) and energy density (ε)
  - Encodes microscopic properties of the theory

Stars



Integrate from the middle until pressure drops below minimum

- Easy
- Explicit Runge–Kutta 4(5) method
- **Adaptive step** size
- Right units

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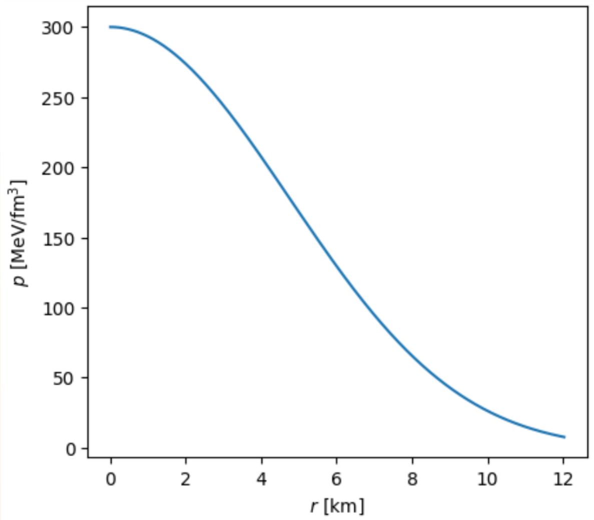
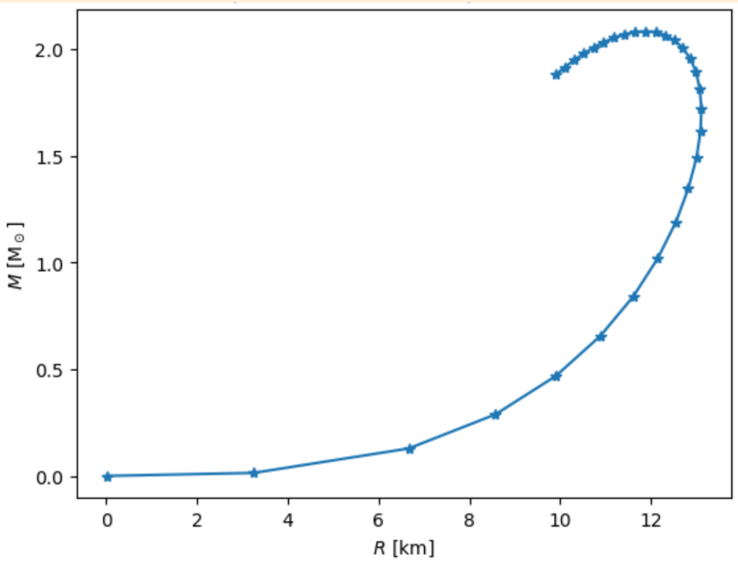


M-R curve:

- Parametrized by **central energy density**
- Each star on the curve is a star



Kaluza–Klein theory

- Gravity + electromagnetism → 5D spacetime
- The 5th dimension is **microscopic**, curled up into a **circle**

  → QM interpretation

Theodor Kaluza (1885-1954)

Oskar Klein (1894-1977)

Base of: scalar-tensor (Brans-Dicke) and string theories

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Theodor Kaluza (1885-1954)

Metric tensor

10 + 4 + 1 independent components

4D metric of gravity

EM vector potential

$$g_{AB} = \begin{bmatrix} g_{\alpha\beta} + \kappa^2\Phi^2 A_\alpha A_\beta & \kappa\Phi^2 A_\alpha \\ \kappa\Phi^2 A_\beta & \Phi^2 \end{bmatrix}$$

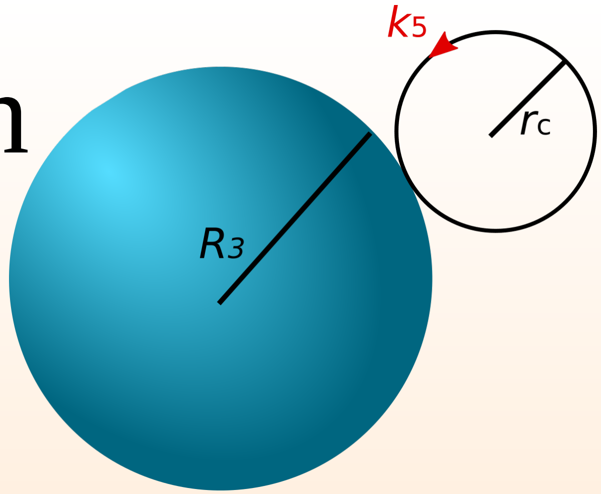
5D metric

Scalar field

Oskar Klein (1894-1977)

Base of: scalar-tensor (Brans-Dicke) and string theories

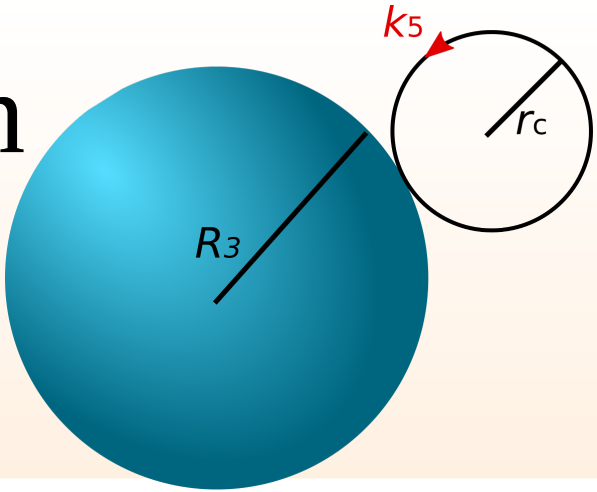
Modified dispersion relation



Particles **moving in** the **extra dimension** possess a modified **effective mass** from a 3-dimensional point of view.

$$E = \sqrt{\mathbf{k}^2 + k_5^2 + m^2} = \sqrt{\mathbf{k}^2 + \left(\frac{N_{\mathrm{exc}}}{r_{\mathrm{c}}}\right)^2 + m^2}$$

Modified dispersion relation



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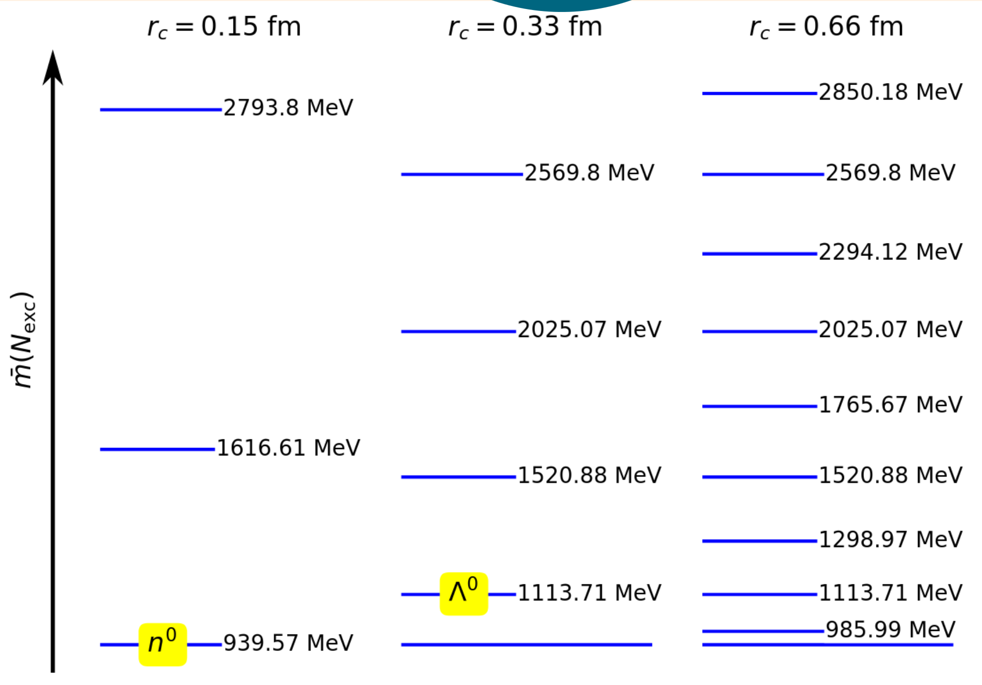
$$\bar{m}^2(N_{\mathrm{exc}}) = m^2 + k_5^2 \qquad k_5 = \frac{N_{\mathrm{exc}}}{r_{\mathrm{c}}}$$



Here the ground state of the Kaluza–
–Klein ladder is the **neutron**.

Equation of state

Thermodynamic potential of a **Fermi gas**:

$$\Omega = -V_{(d)} \sum_{i=0}^{N_{\rm exc}} \frac{g_i}{\beta} \int \frac{\mathrm{d}^d \mathbf{k}}{(2\pi)^d} \times \times \left[\ln\left(1+\mathrm{e}^{-\beta(E_i-\mu)}\right) + \ln\left(1+\mathrm{e}^{-\beta(E_i+\mu)}\right)\right]$$

Zero temperature approximation:

$$T \ln\left(1+\mathrm{e}^{-(E-\mu)/T}\right)\Big|_{T=0} = \begin{cases} \mu - E, & \text{if } E < \mu \\ 0, & \text{if } E \geq \mu \end{cases}$$

Repulsive **potential**:

$$U(n) = \xi n$$

J. Zimanyi, B. Lukacs, P. Levai, J.P. Bondorf: „An Interpretable Family ofEquation of State for Dense Hadronic Matter", Nucl.Phys. A484 (1988) 647

State variables:

$$p(\mu) = p_0(\bar{\mu}) + p_{\rm int}$$

$$\varepsilon(\mu) = \varepsilon_0(\bar{\mu}) + \varepsilon_{\rm int}$$

$$\bar{\mu} = \mu - U(n)$$

$$n(\bar{\mu}) = n_0(\bar{\mu})$$

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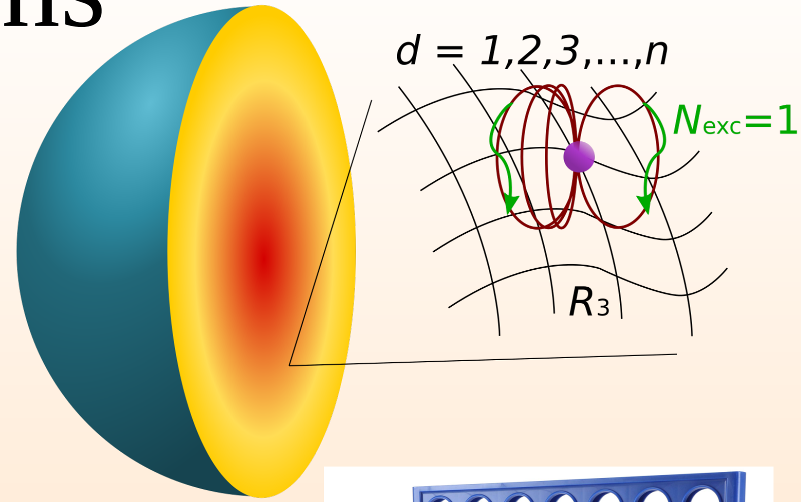
Multiple extra dimensions



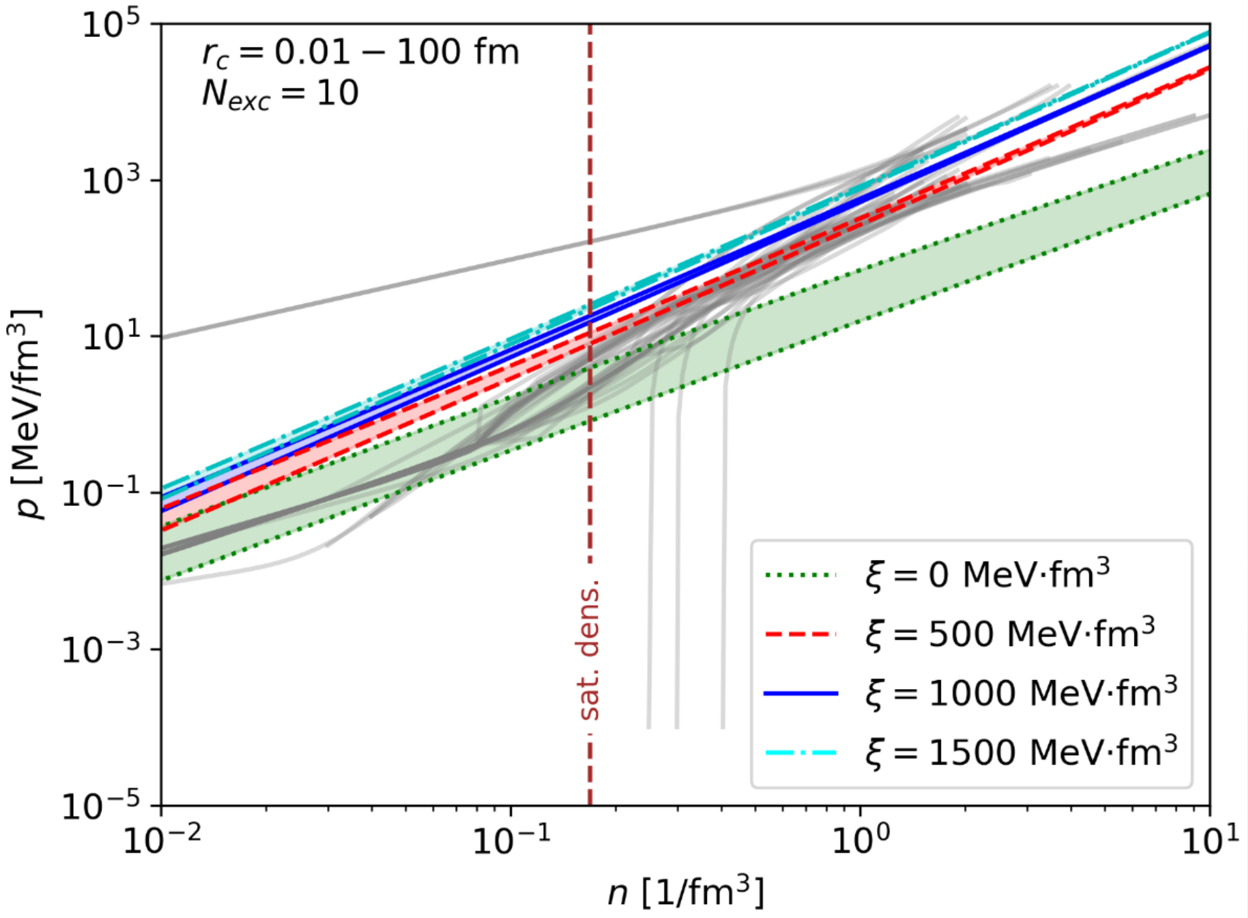
- All extra dimensions are the **same size**
- Particles are allowed to have upmost **one excitations** in all extra dimensions
- Effective mass:

$$\tilde{m}^2 = m^2 + \sum_{j=1}^{d} \tilde{k}_j^2$$

- Thermodynamic potential:

$$\Omega = -V_{(3+d)} \sum_i \underbrace{\sum_{j=0}^{N_{\mathrm{exc}}} \cdots \sum_{l=0}^{N_{\mathrm{exc}}}}_{d} \frac{g_i}{\beta} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \times$$
$$\times \left[\ln\left(1 + \mathrm{e}^{-\beta(E_{ij\ldots l} - \mu)}\right) + \ln\left(1 + \mathrm{e}^{-\beta(E_{ij\ldots l} + \mu)}\right) \right]$$

Equation of state

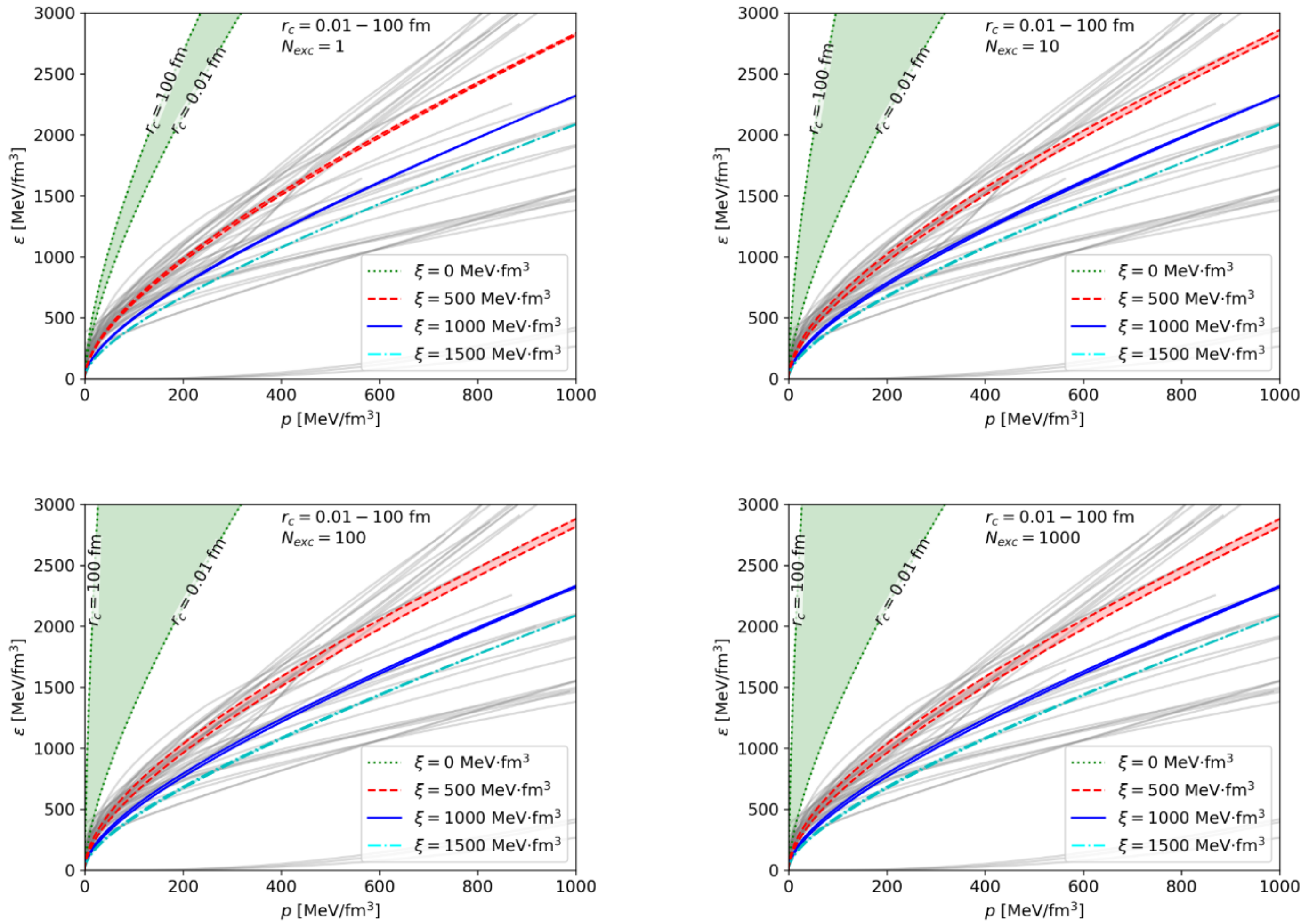


Pressure as a function of baryon number density.

F. Özel and P. Freire, "Masses, Radii, and the Equation of State of Neutron Stars," Ann. Rev. Astron. Astrophys. **54** (2016), 401-440 doi:10.1146/annurev-astro-081915-023322 [arXiv:1603.02698 [astro-ph.HE]].

https://compose.obspm.fr/

Equation of state



Energy density as a function of pressure.

F. Özel and P. Freire, "Masses, Radii, and the Equation of State of Neutron Stars," Ann. Rev. Astron. Astrophys. **54** (2016), 401-440 doi:10.1146/annurev-astro-081915-023322 [arXiv:1603.02698 [astro-ph.HE]].

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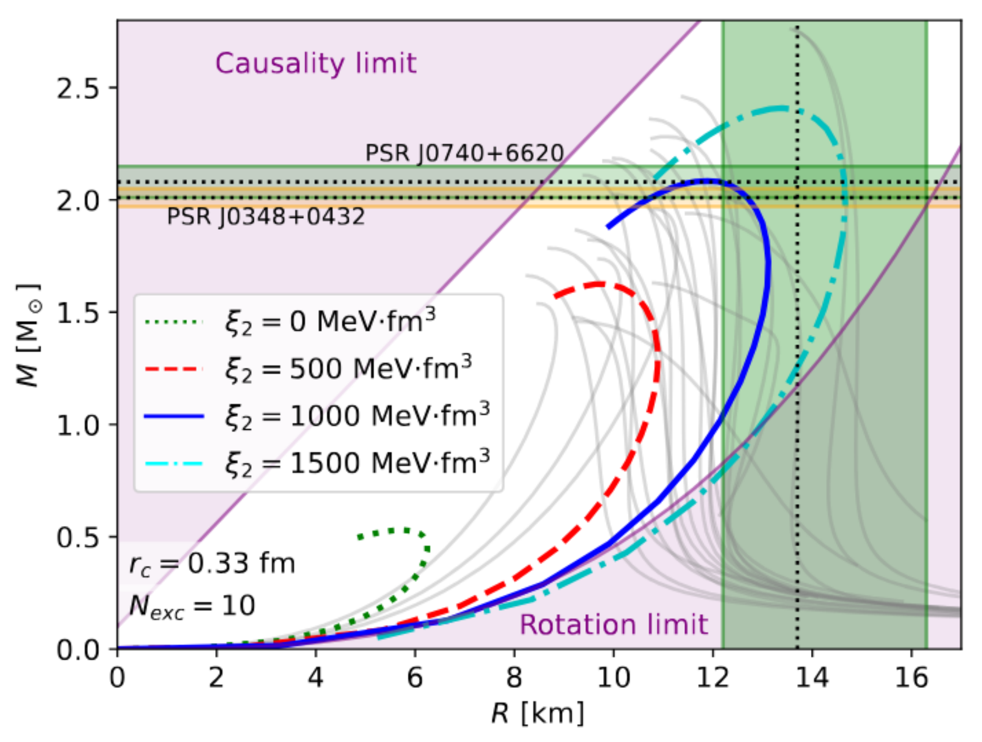
Comparison to observation



Fonseca E., et al., 2021, The Astrophysical Journal Letters, 915, L12
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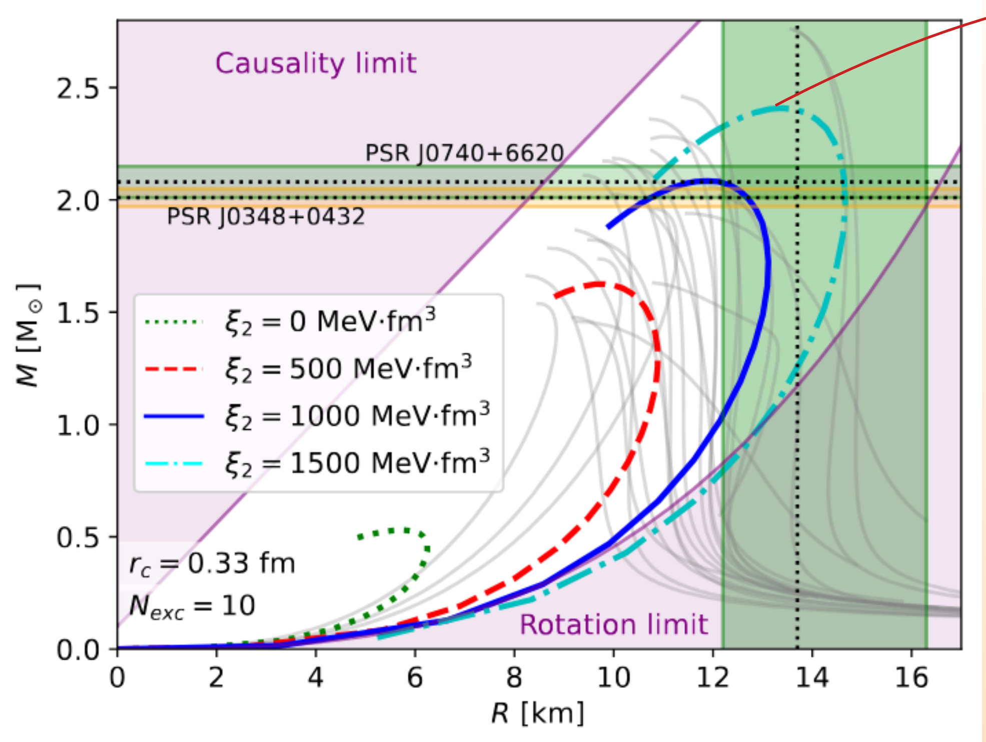

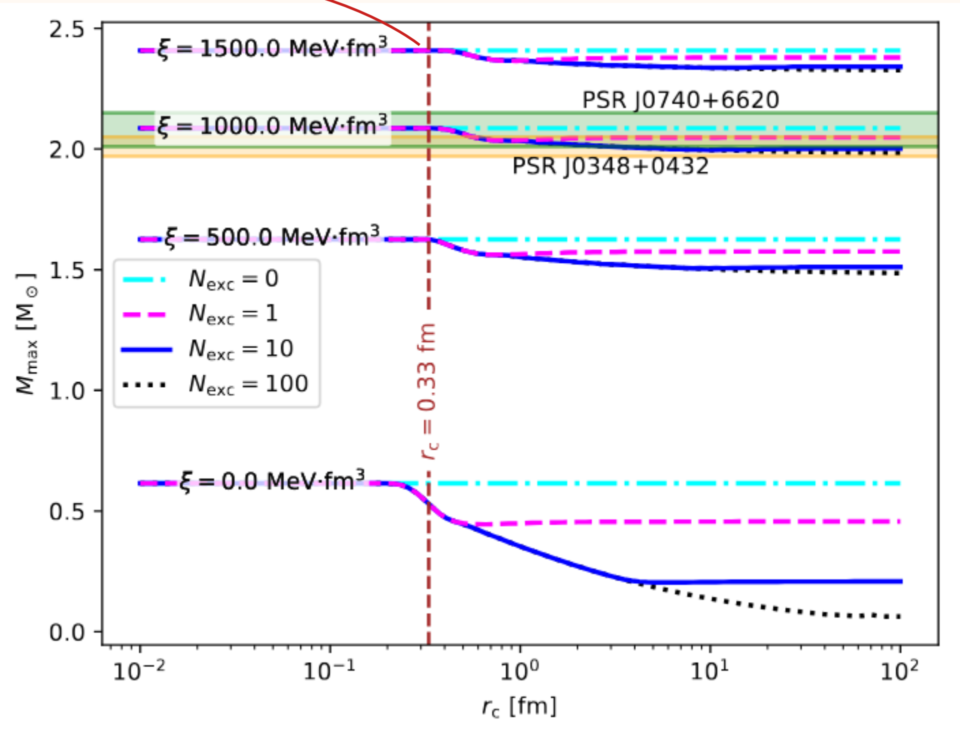

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Extra-dimensional theories

Observations regarding **neutron stars** can be **relevant for constraining** multiple beyond standard model extra-dimensional theories.

- Randall–Sundrum, GUT $r_c < 10^{-17}$ m
- ADD large extra dimensions (TeV scale) $r_c < 1.9\times10^{-4}$ m
- Precision measurements (tabletop) $r_c < 8.0\times10^{-5}$ m
- Astrophysics – gravitational waves $r_c < 10^{-6}$ m

Randall L., Sundrum R., 1999, Phys. Rev. Lett., 83, 3370
Cheung K., Landsberg G., 2002, Physical Review D, 65
Bernardi G., 2003. https://api.semanticscholar.org/CorpusID: 121772649
Abdallah J., et al., 2009, Eur. Phys. J. C, 60, 17
Adelberger E. G., Gundlach J. H., Heckel B. R., Hoedl S., Schlamminger S., 2009, Prog. Part. Nucl. Phys., 62, 102
Eötvös R. V., Pekár D., Fekete E., 1922, Annalen Phys., 68, 11
Péter G., Deák L., Gróf G., Kiss B., Szondy G., Tóth G., Ván P., Völgyesi L., 2022, Repeating the Eötvös-Pekár-Fekete equivalence principle measure-ments (arXiv:2205.14587), https://arxiv.org/abs/2205.14587
Murata J., Tanaka S., 2015, Class. Quant. Grav., 32, 033001
Abbott R., et al., 2021, Tests of General Relativity with GWTC-3 (arXiv:2112.06861)

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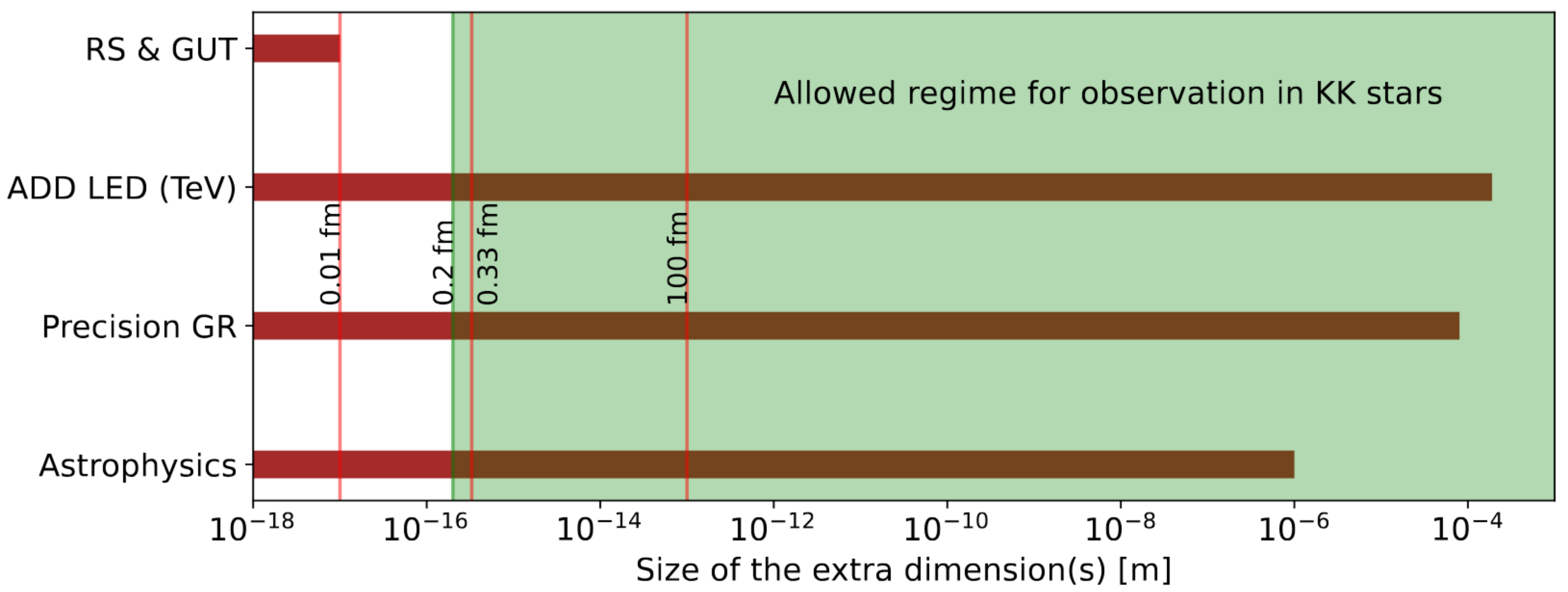
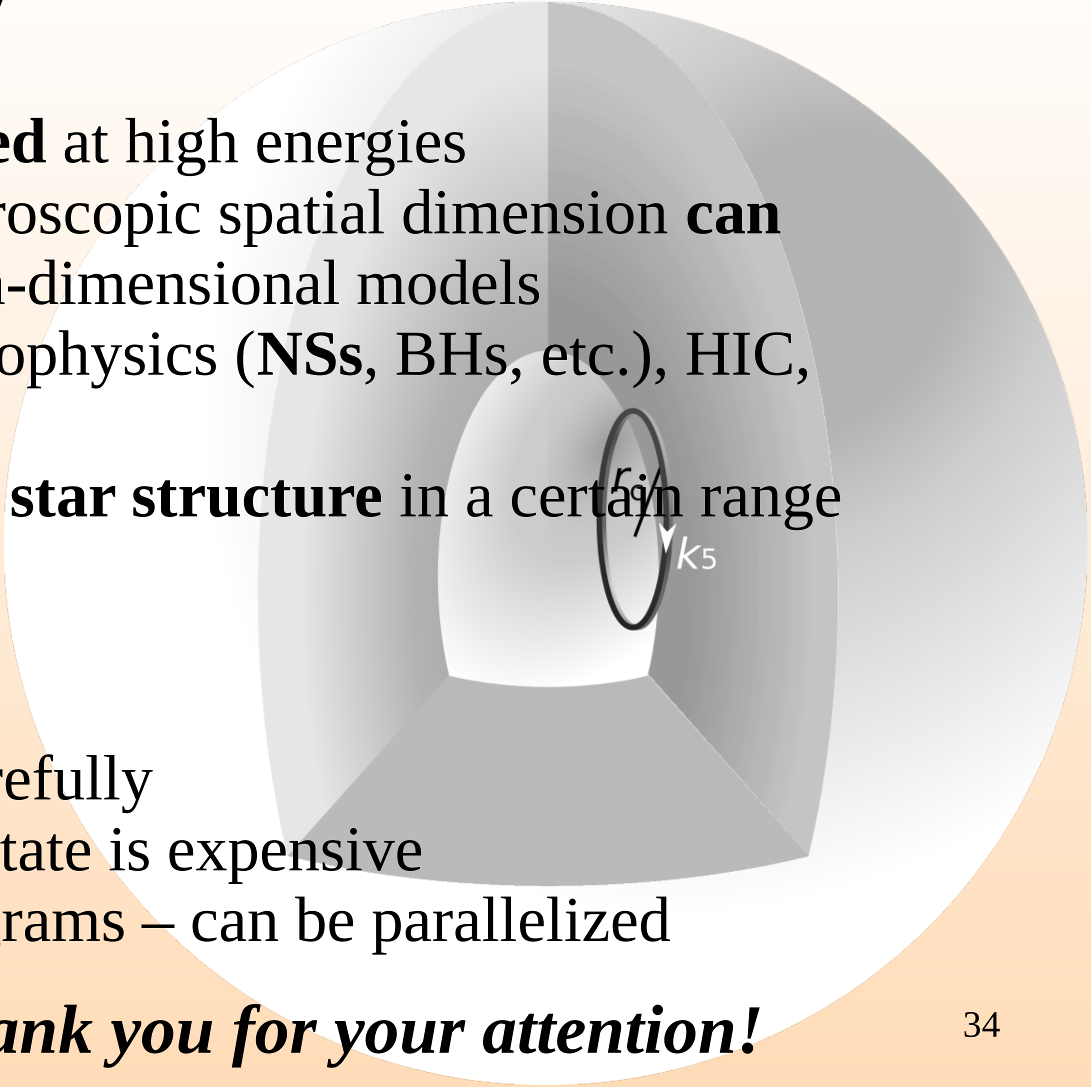


Summary

- **New theories** of physics are **needed** at high energies
- **Kaluza–Klein** with one extra microscopic spatial dimension **can show** the **phenomenology** of extra-dimensional models
- **Observational** possibilities in astrophysics (**NSs**, BHs, etc.), HIC, tabletop
- **Extra dimensions affect** compact **star structure** in a certain range

$$r_c \gtrsim 0.2 \text{ fm}$$

- **Numerical difficulties**:
  - integrator must be chosen carefully
  - determining the equation of state is expensive
  - calculating multiple MR diagrams – can be parallelized



Thank you for your attention!

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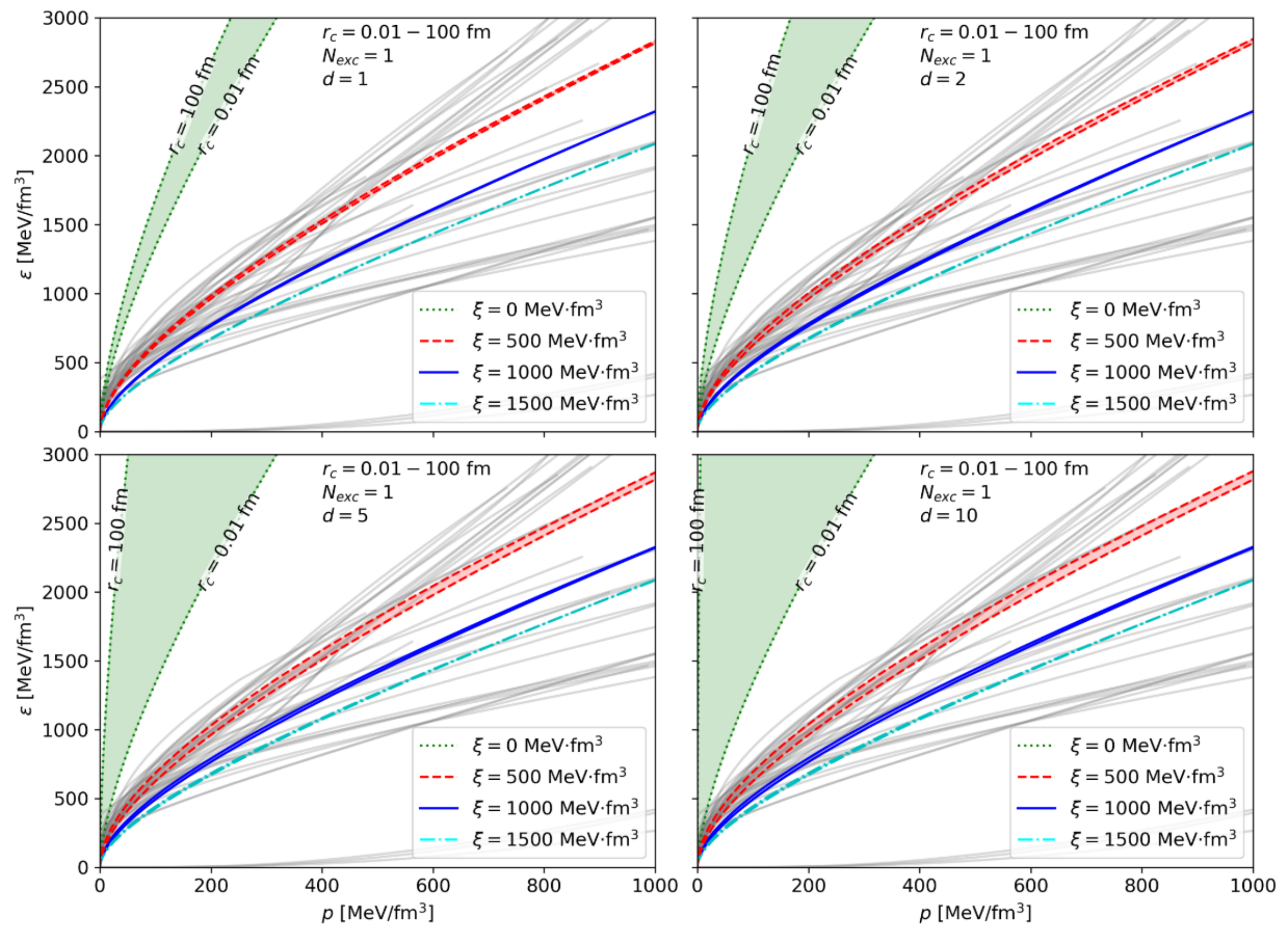


Modified phasespace

In collaboration with:
Aneta Wojnar

- Strong gravitational field could affect microscopic physics
- Relevance for the structure of NSs, BHs, WDs, planets?
- Modification to particle paths?
- Modified thermodynamics?

- Generalized uncertainty principle (GUP)

$$\Delta x \Delta p \geq \frac{\hbar}{2} + \beta \Delta p^2$$

- In terms of the curvature

$$\sigma_p \rho \gtrsim \pi\hbar \left[1 - \frac{\rho^2 \mathcal{R}|_{p_0}}{12\pi^2} + \xi \frac{\rho^4}{\lambda_C^2} \nabla_j N_i \nabla^j N^i |_{p_0} \right]$$

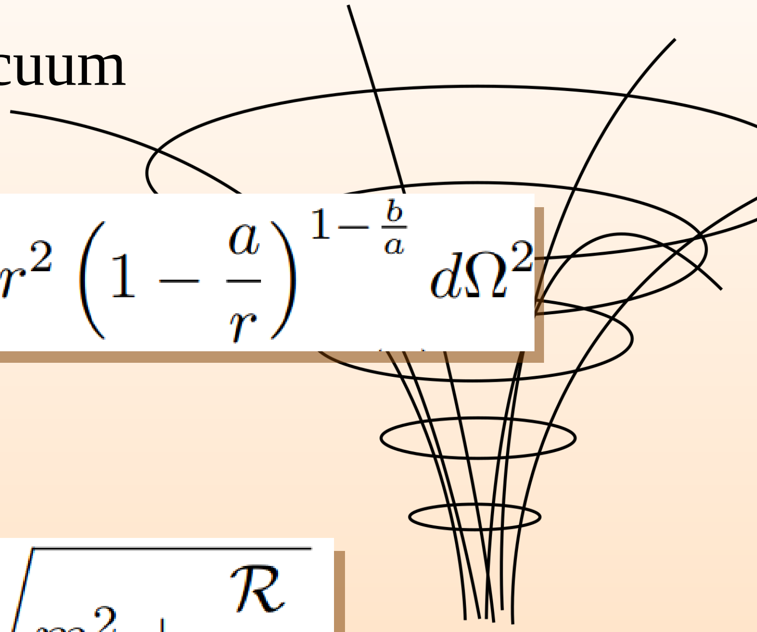
L. Petruzziello and F. Wagner, Physical Review D 103, 104061 (2021).
M. P. Dabrowski and F. Wagner, The European Physical Journal C 80, 676 (2020).
Abdel Nasser Tawfik and Abdel Magied Diab 2015 Rep. Prog. Phys. 78 126001
Aleksander Kozak, Aneta Wojnar, “Earthquakes as probing tools for gravity theories”, 2023, arXiv:2308.01784

Schwarzschild-like solution

In collaboration with:
Aneta Wojnar

- g_{55} is allowed to vary ⟶ scalar field
- Non-zero energy-momentum tensor even for vacuum
- Schwarzschild-like metric

$$ds^2 = -\left(1-\frac{a}{r}\right)^{\frac{b}{a}} dt^2 + \left(1-\frac{a}{r}\right)^{-\frac{b}{a}} dr^2 + r^2 \left(1-\frac{a}{r}\right)^{1-\frac{b}{a}} d\Omega^2$$



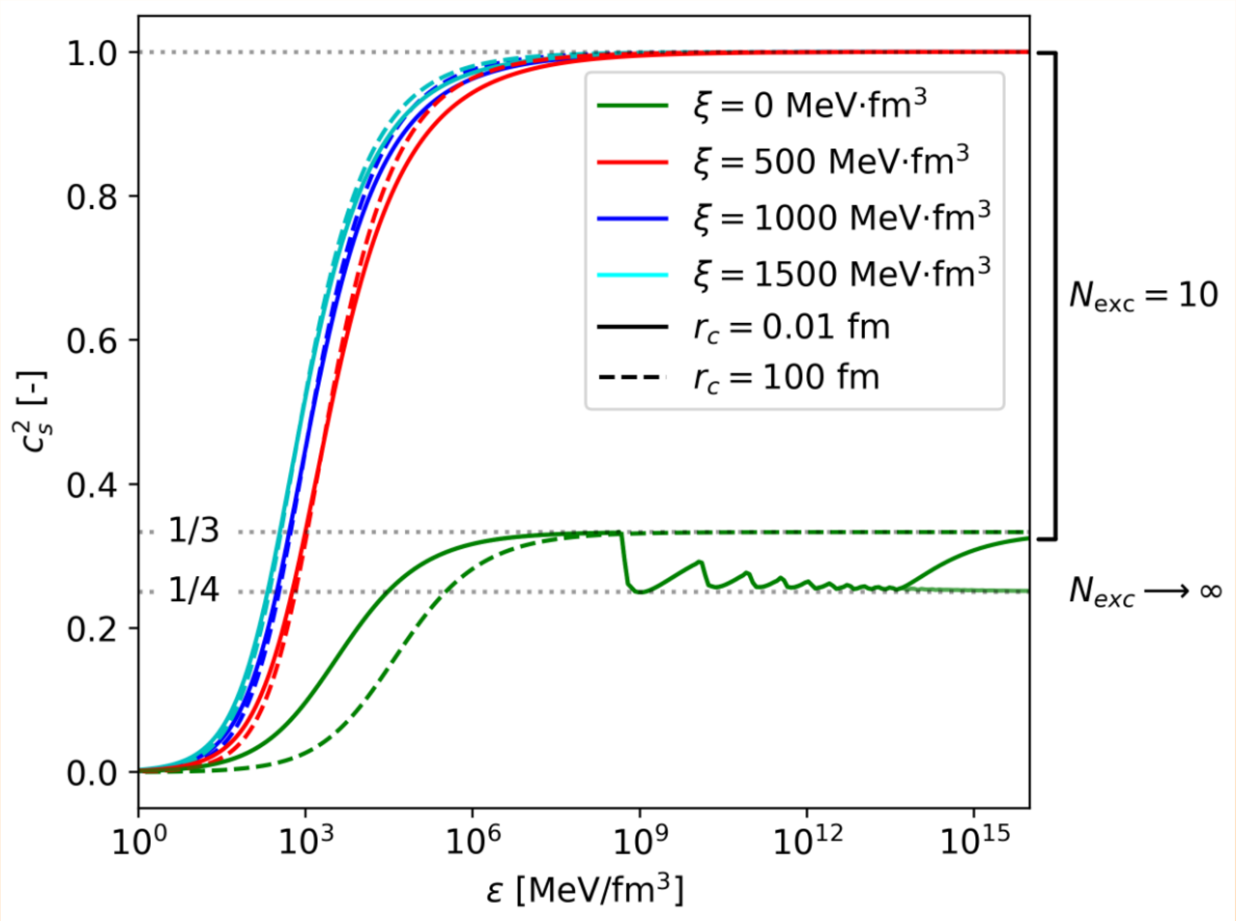
- Non-zero phase-space and spacetime curvatures
- Modified dispersion relation and effective mass

$$p^\mu p_\mu = -m^2 c^2 - \frac{\mathcal{R}}{6}$$

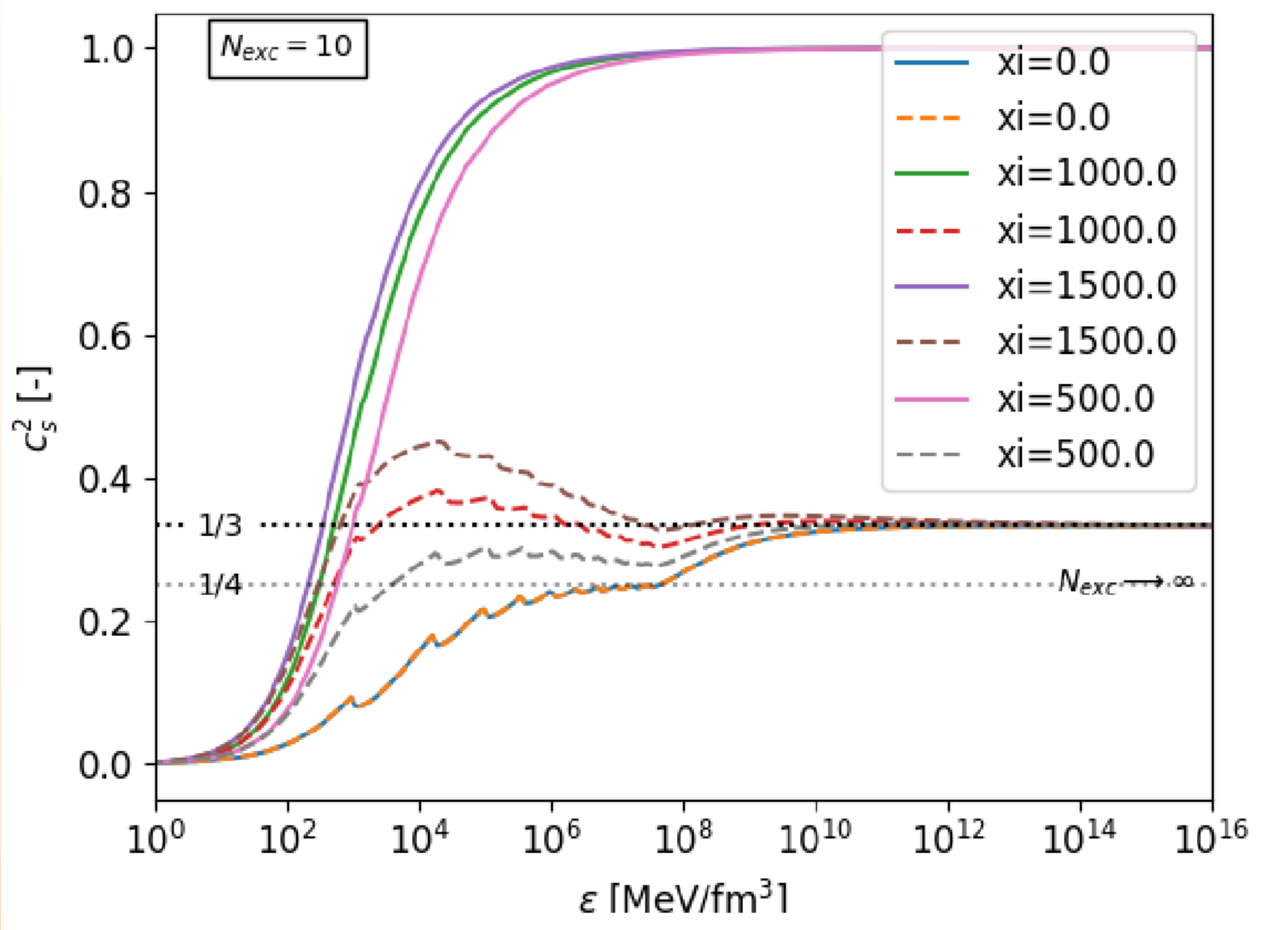
$$m_{\text{eff}} = \sqrt{m^2 + \frac{\mathcal{R}}{6c^2}}$$

R. Coquereaux and G. Esposito-Farese, in Annales de l'IHP Physique th´eorique, Vol. 52 (1990) pp. 113–150
L. Petruzziello and F. Wagner, Physical Review D 103, 104061 (2021).
M. P. Dabrowski and F. Wagner, The European Physical Journal C 80, 676 (2020).

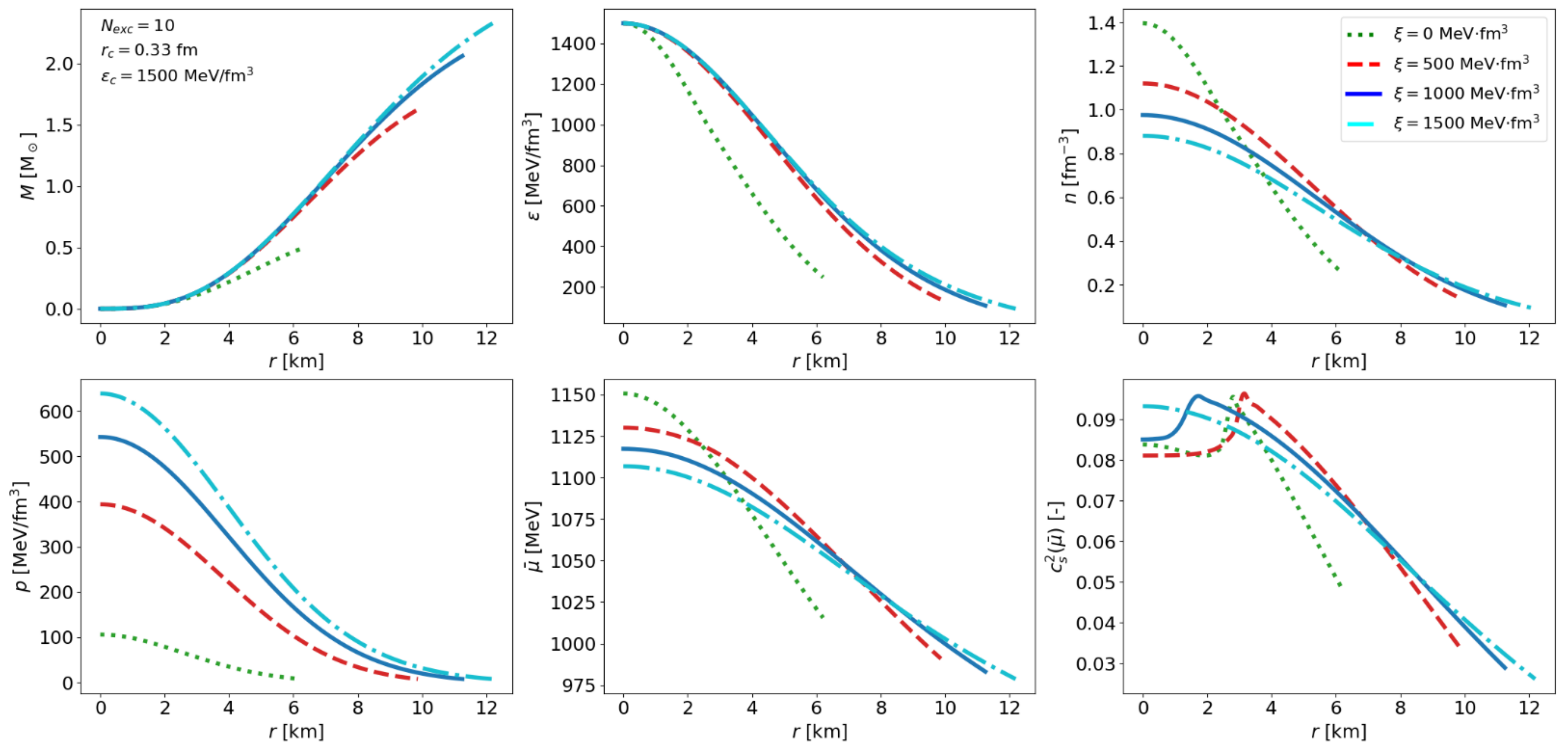
Speed of sound



Speed of sound – modified potential



Solving the TOV equation – stars



r: radius of star