Cosmological N-body Simulations of Rotating Universes

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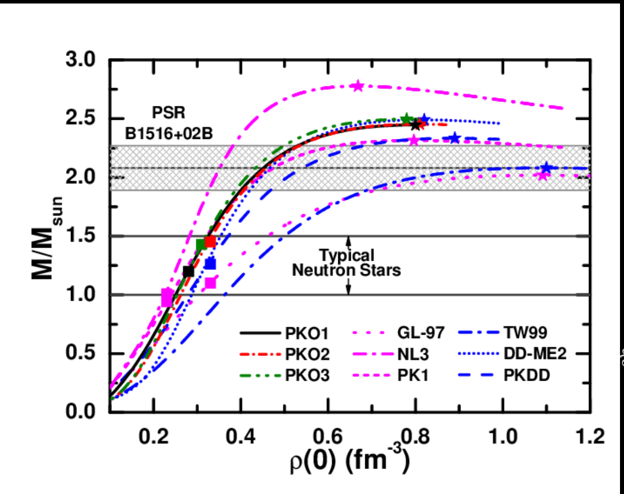
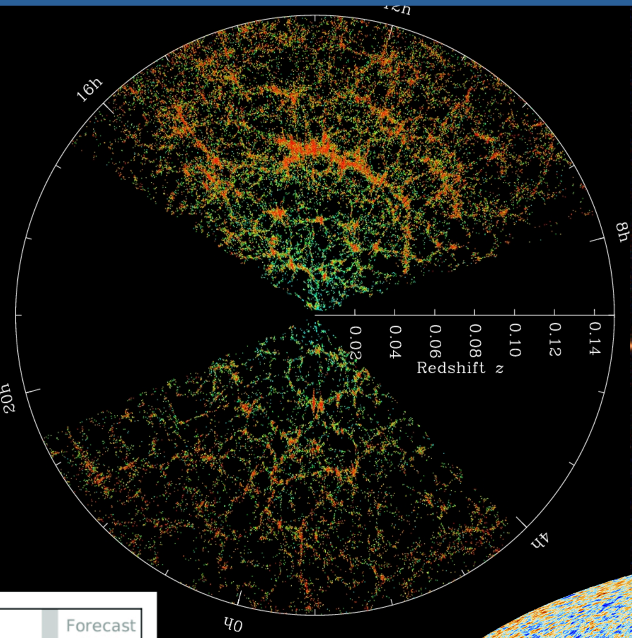
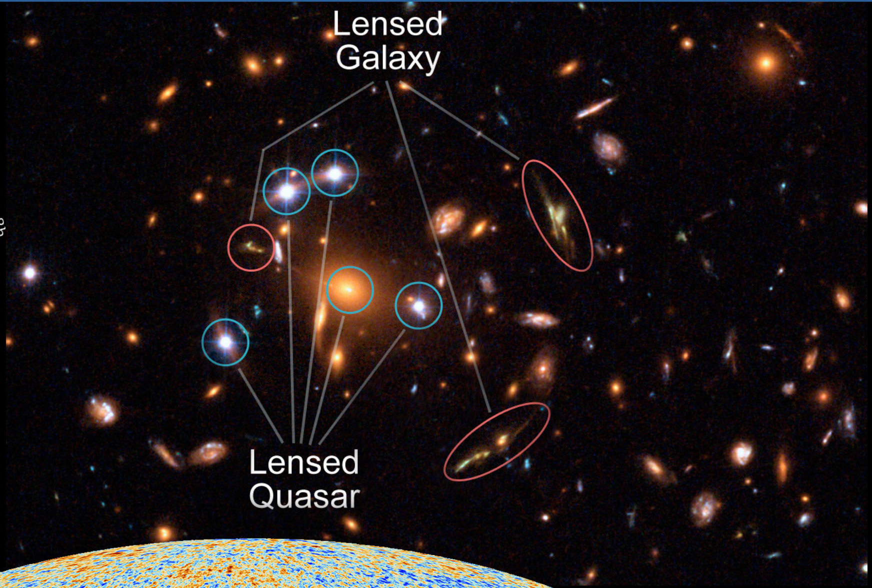
[4]*Institute for Astronomy*, **University of Hawai'i**

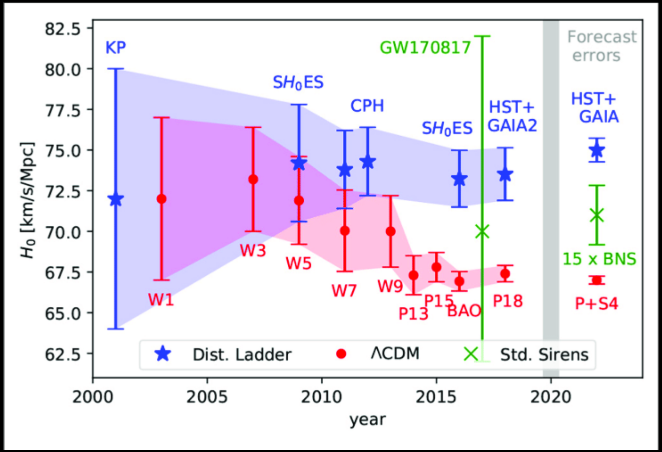
XV. GPU Day – 2025
HUN-REN Center

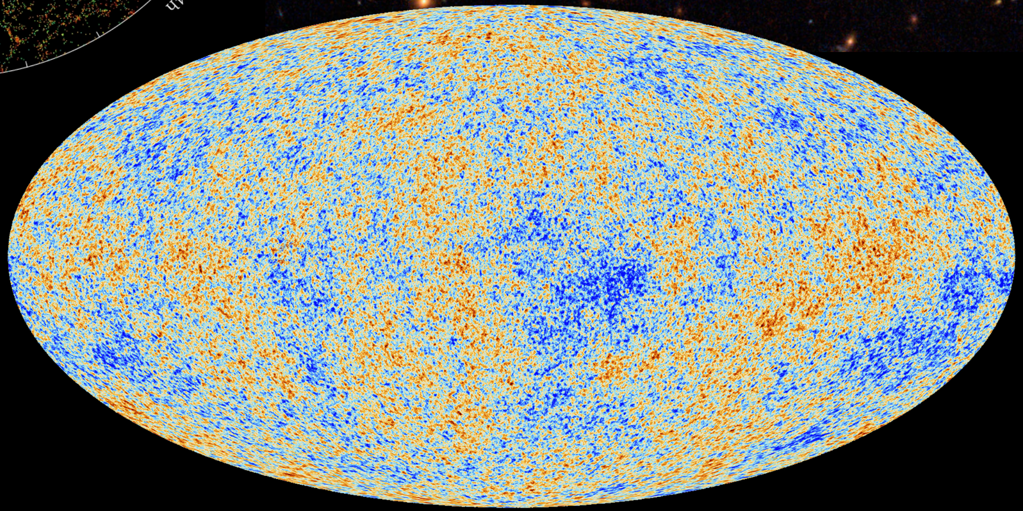
Cosmology nowadays is a mess











One of the interesting ideas

Balázs E. Szigeti, István Szapudi,
Imre F. Barna, Gergely G. Barnaföldi (2025)

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- *Rotating Universe?* **G. Gamov** (1946)

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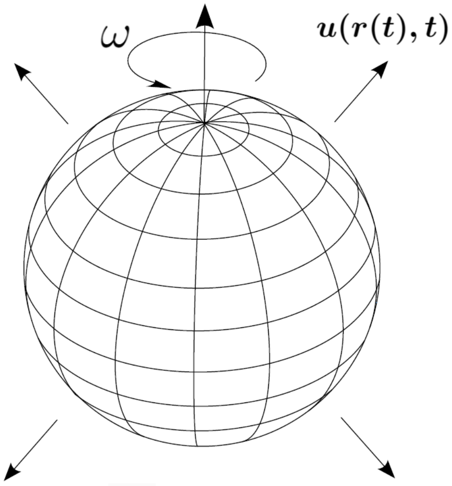

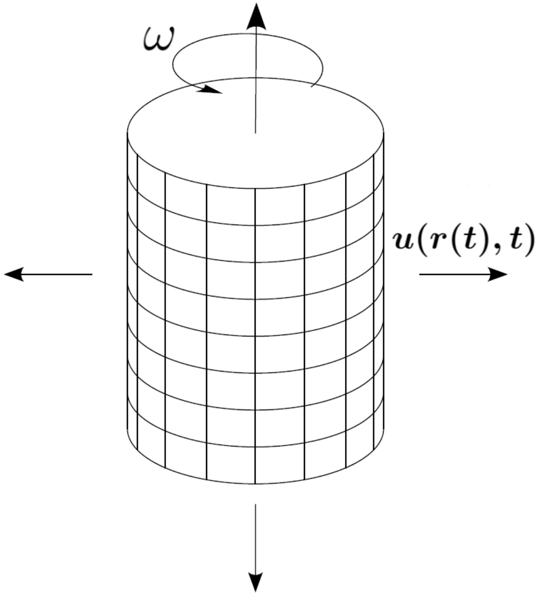

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Can Rotation Solve the Hubble Puzzle?

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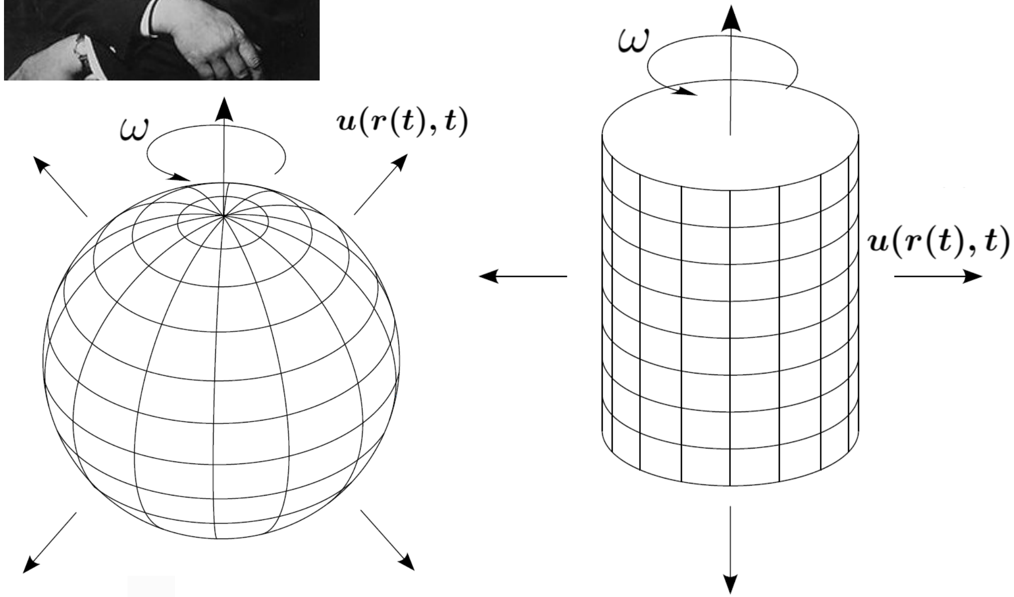

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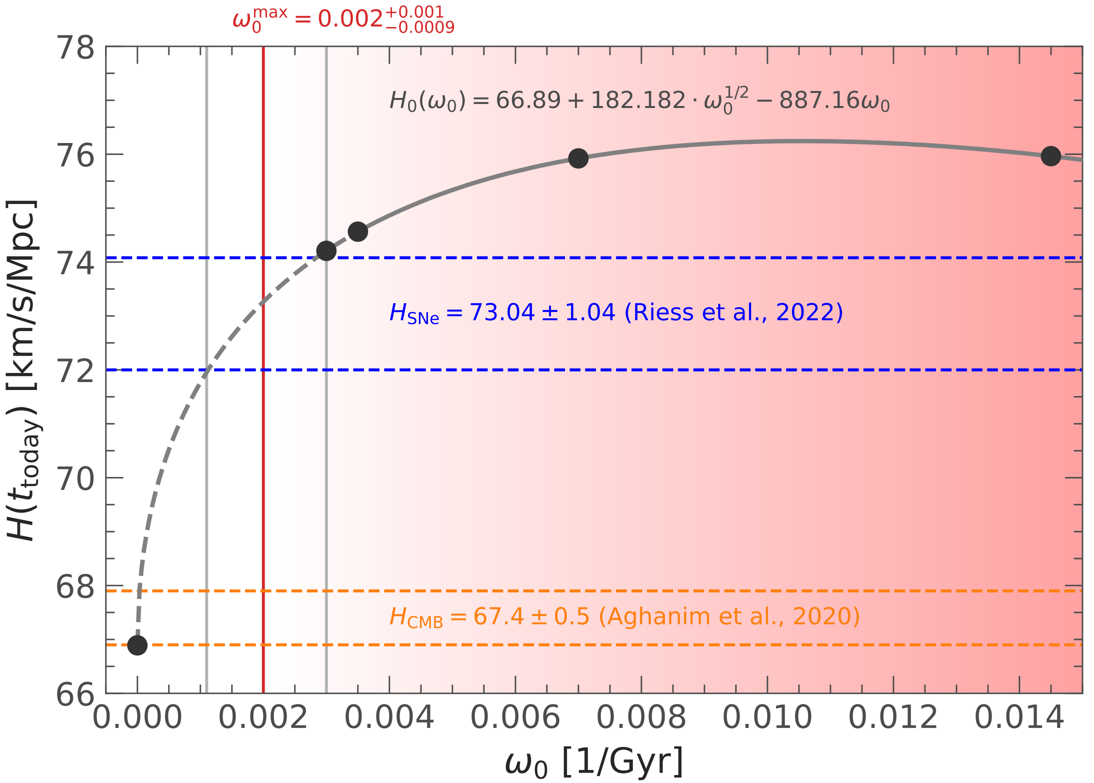

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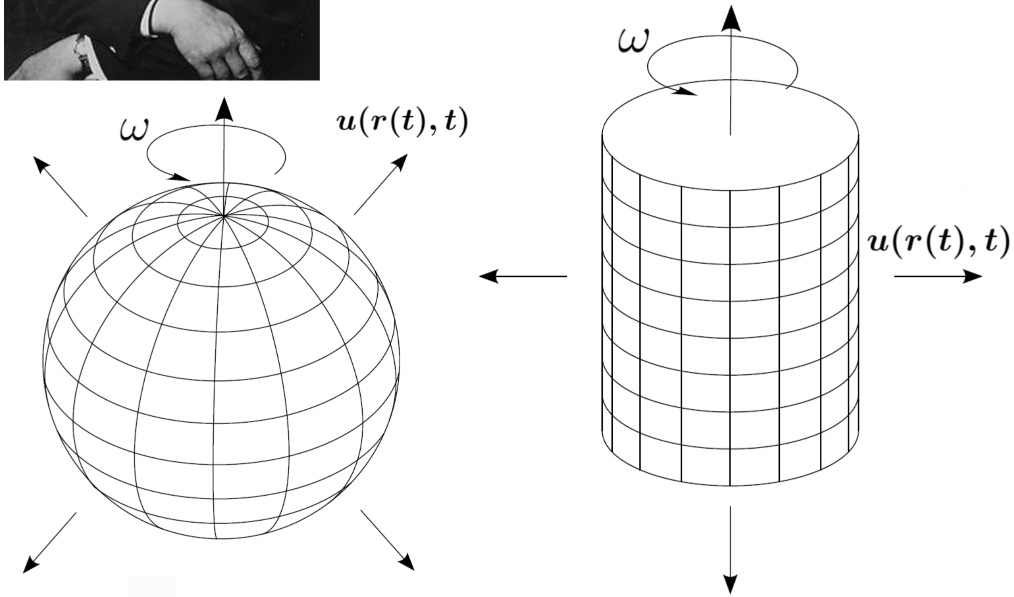

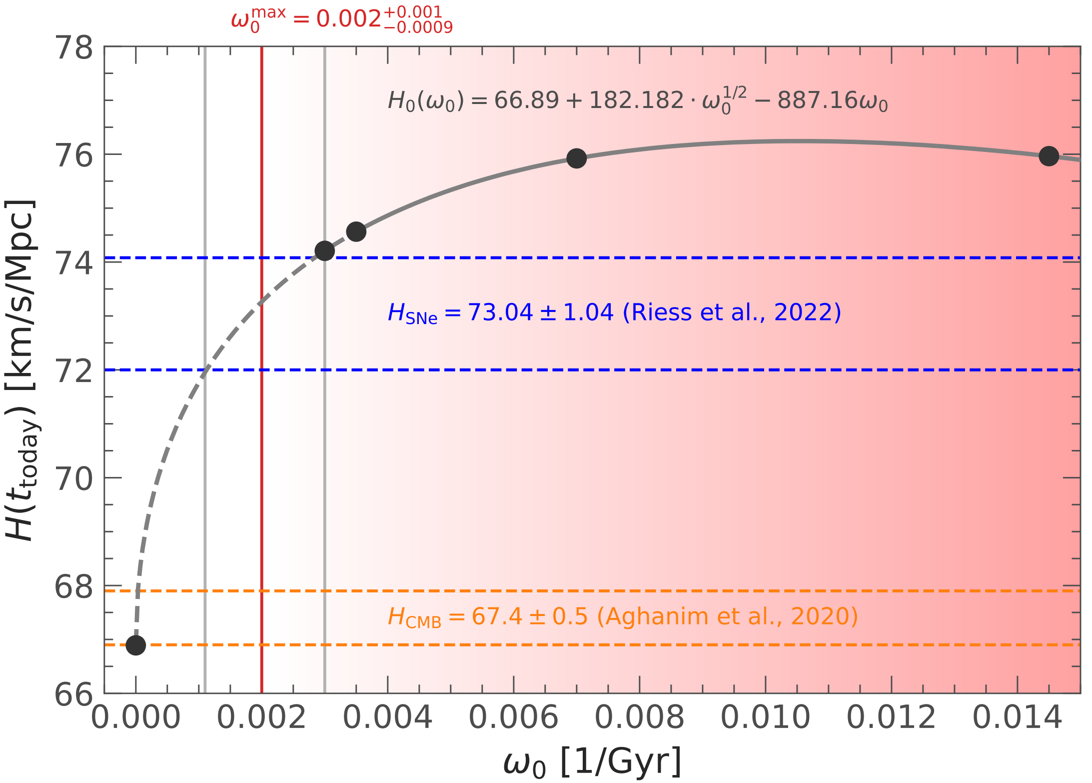

"But where are the simulations?"

What even are cosmological simulations?

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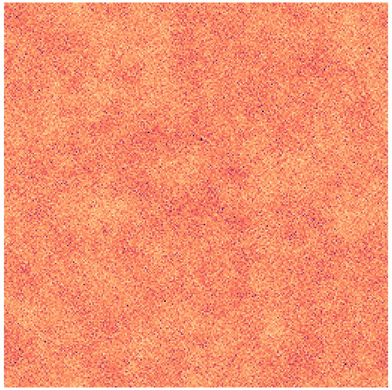
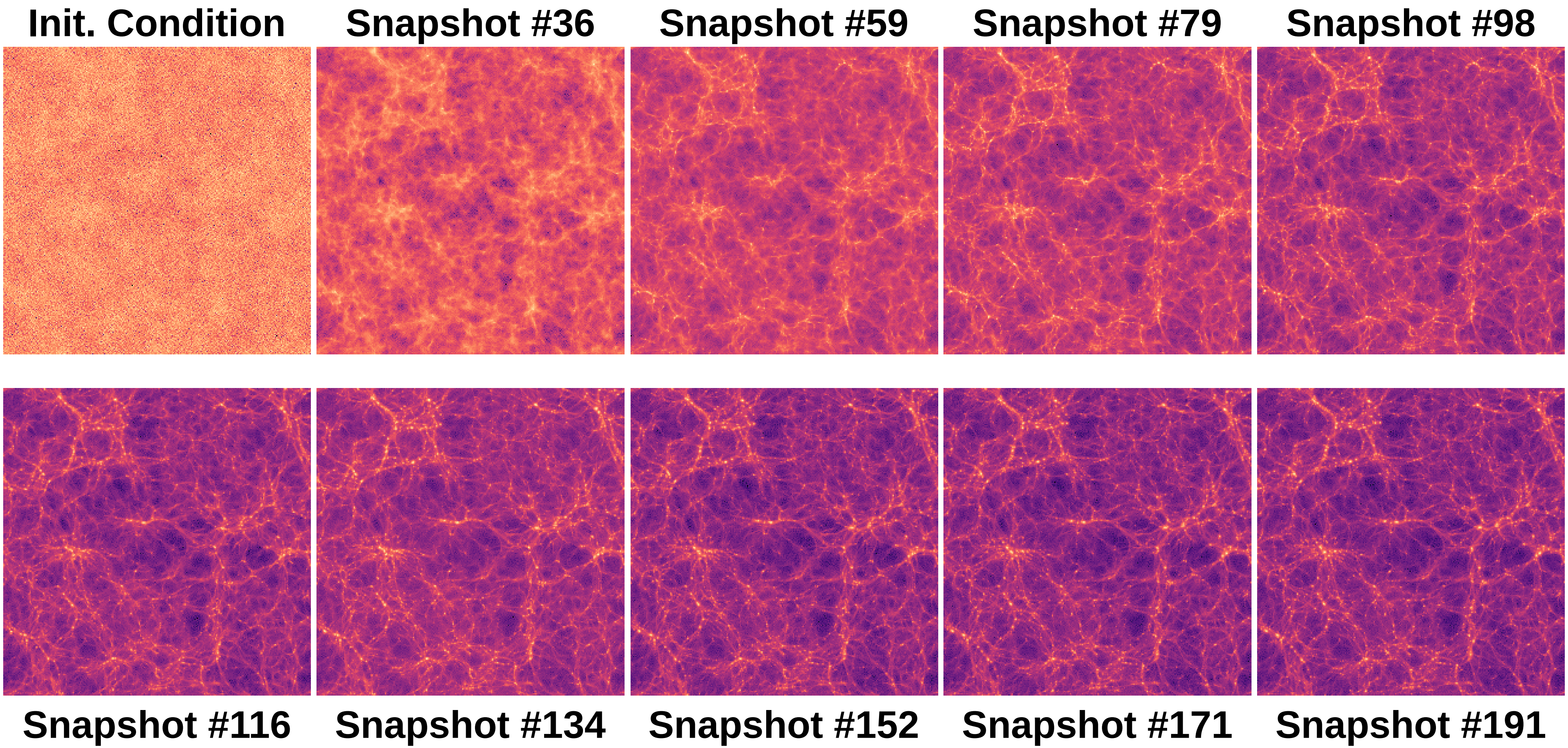
Init. Condition



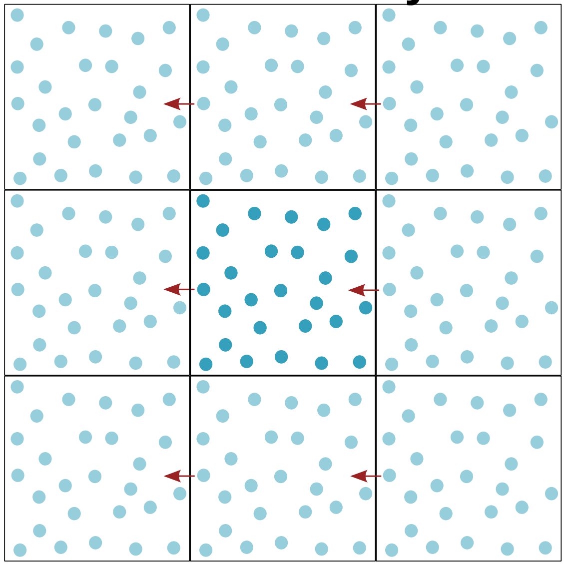
What even are cosmological simulations?

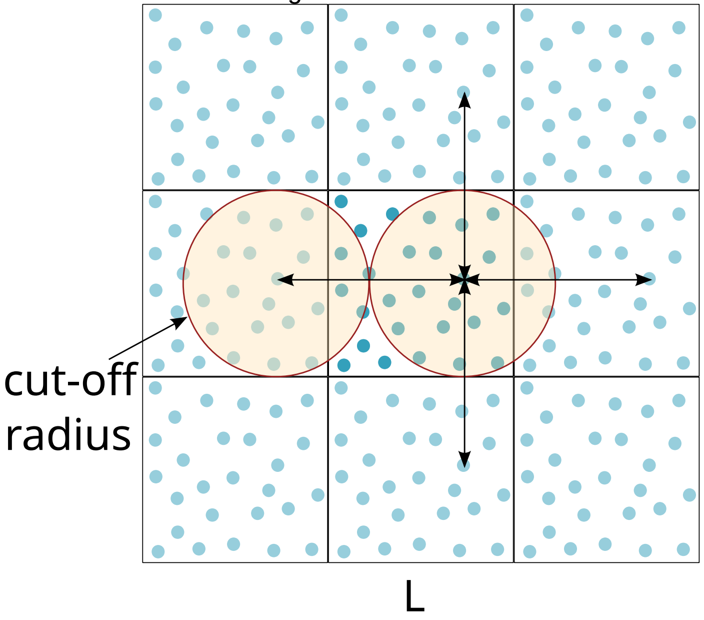


Rethinking the boundary conditions

Periodic boundary conditions

Source: Digital Research Alliance of Canada





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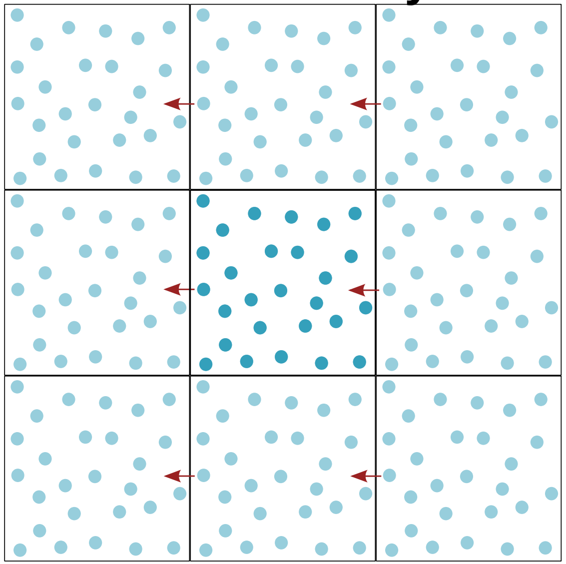

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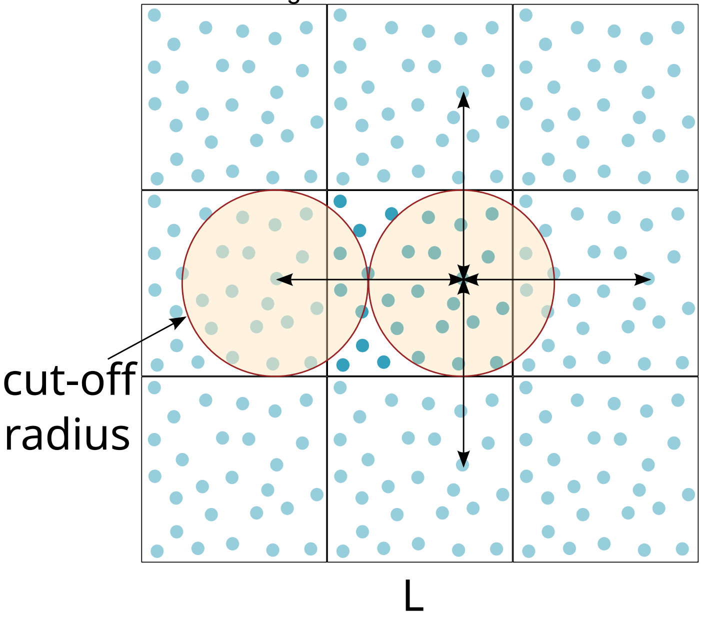
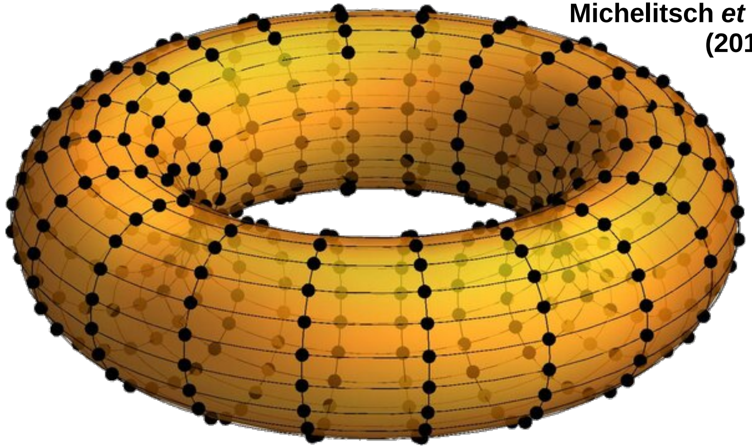


Michelitsch *et al.* (2017)



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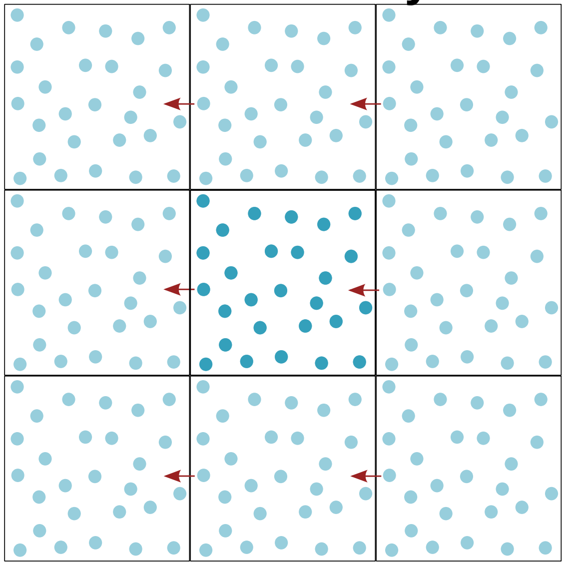

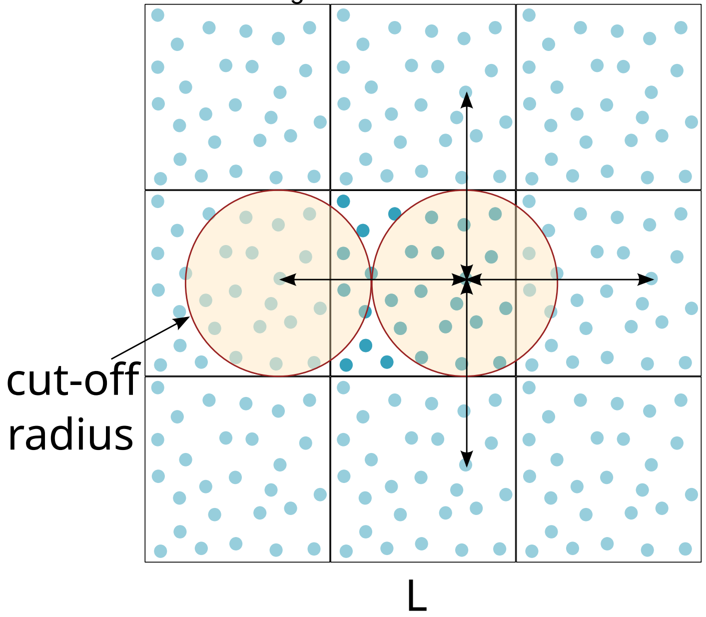

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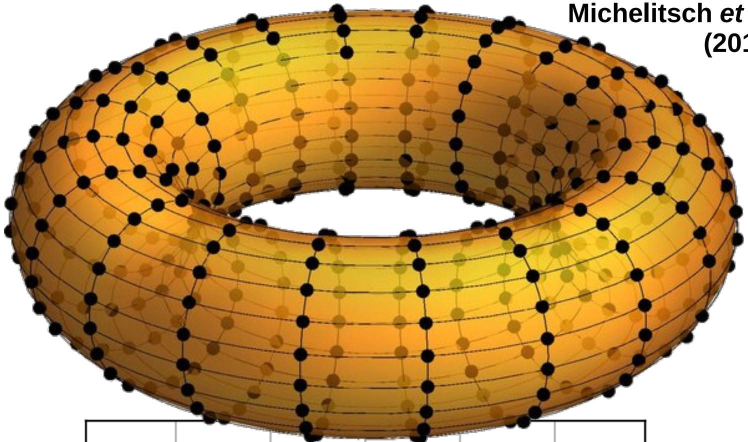
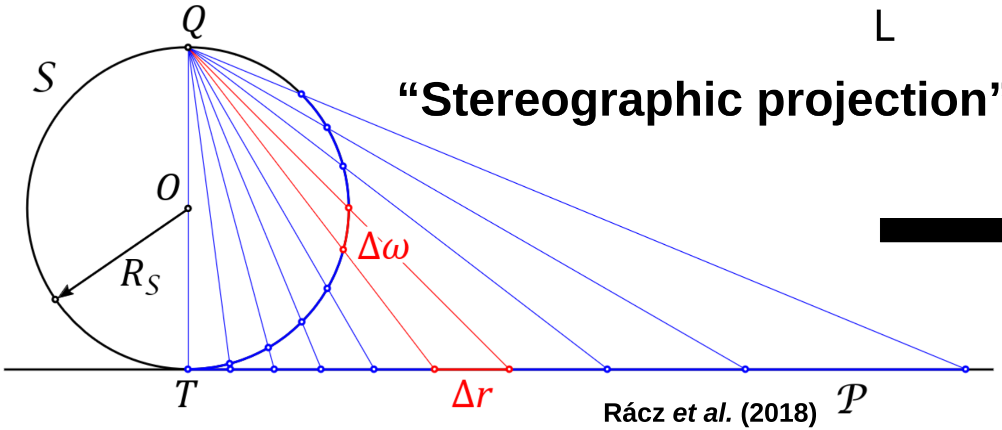
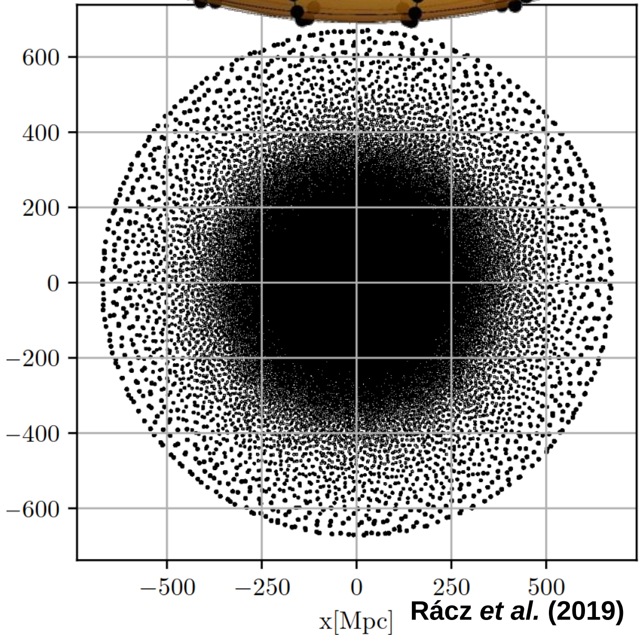




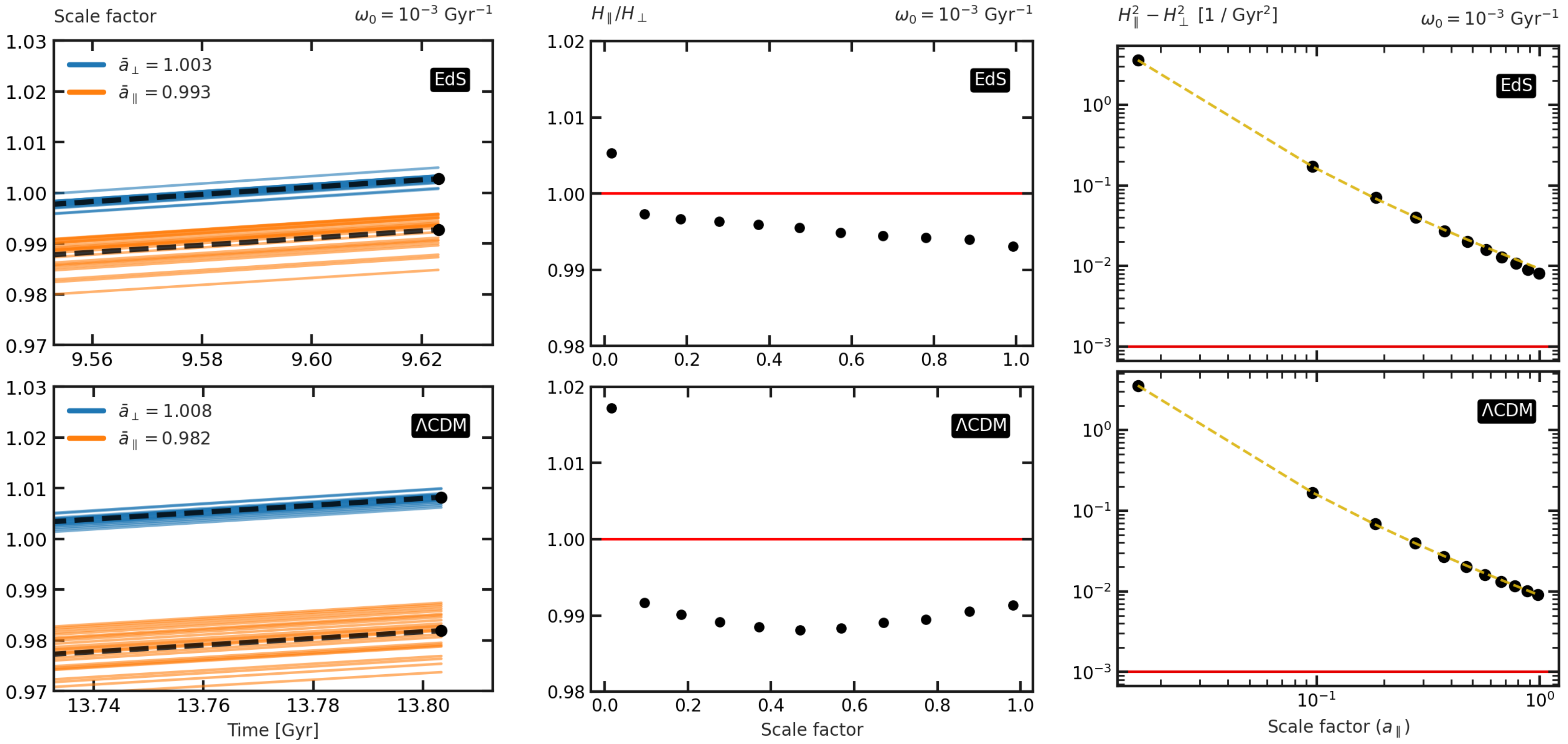
"Stereographic projection"

Rácz *et al.* (2018)



Rácz *et al.* (2019)

Results of spherical simulations



***Simulating Rotating Newtonian Universes* – Pál *et. al.* (2025)**

Challenges of cylindrical simulations

Challenges of cylindrical simulations

Gravitational force of a **sphere**

Gravitational force of a **cylinder**

Challenges of cylindrical simulations

Gravitational force of a **sphere**

$$\mathbf{F}(r) = \begin{cases} -\dfrac{GMm}{r^2}\hat{\mathbf{r}}, & r \geq R, \\ -\dfrac{GMm}{R^3}\, r\hat{\mathbf{r}}, & 0 \leq r \leq R. \end{cases}$$

Gravitational force of a **cylinder**

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Summ

$$F_h = \begin{cases} -2G\rho \displaystyle\int_0^R \ln\left(\frac{[L^2+R^2+a^2+2a(R^2-Y^2)^{1/2}]^{1/2}+L}{[L^2+R^2+a^2-2a(R^2-Y^2)^{1/2}]^{1/2}+L}\right)\mathrm{d}Y \\ \quad + G\rho \displaystyle\int_0^R \ln\left(\frac{R^2+a^2+2a(R^2-Y^2)^{1/2}}{R^2+a^2-2a(R^2-Y^2)^{1/2}}\right)\mathrm{d}Y \qquad (a \neq R) \\ -2G\rho \displaystyle\int_0^R \ln\left(\frac{[L^2+2R^2+2R(R^2-Y^2)^{1/2}]^{1/2}+L}{[L^2+2R^2-2R(R^2-Y^2)^{1/2}]^{1/2}+L}\right)\mathrm{d}Y + \pi G\rho R \qquad (a = R) \end{cases} \tag{18}$$

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Additional problems

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- Mixed boundary conditions
- Ewald summation is necessary, as the simulation otherwise collapses due to numerical effects

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- Ewald summation is necessary, as the simulation otherwise collapses due to numerical effects
- Ewald summation necessitates the force calculation for a finite cylinder instead of an infinite

Gravitational force of a **cylinder**

Summ

$$F_h = \begin{cases} -2G\rho \displaystyle\int_0^R \ln\left(\frac{[L^2+R^2+a^2+2a(R^2-Y^2)^{1/2}]^{1/2}+L}{[L^2+R^2+a^2-2a(R^2-Y^2)^{1/2}]^{1/2}+L}\right) \mathrm{d}Y \\ \quad + G\rho \displaystyle\int_0^R \ln\left(\frac{R^2+a^2+2a(R^2-Y^2)^{1/2}}{R^2+a^2-2a(R^2-Y^2)^{1/2}}\right) \mathrm{d}Y \qquad (a \neq R) \\ -2G\rho \displaystyle\int_0^R \ln\left(\frac{[L^2+2R^2+2R(R^2-Y^2)^{1/2}]^{1/2}+L}{[L^2+2R^2-2R(R^2-Y^2)^{1/2}]^{1/2}+L}\right) \mathrm{d}Y + \pi G\rho R \qquad (a = R) \end{cases} \tag{18}$$

$$F_v = \begin{cases} 2G\rho \displaystyle\int_0^R \ln\left(\frac{[L^2+R^2+a^2+2a(R^2-Y^2)^{1/2}]^{1/2}-(R^2-Y^2)^{1/2}-a}{[L^2+R^2+a^2-2a(R^2-Y^2)^{1/2}]^{1/2}+(R^2-Y^2)^{1/2}-a}\right) \mathrm{d}Y \\ \quad +2G\rho \displaystyle\int_0^R \ln\left(\frac{[R^2+a^2+2a(R^2-Y^2)^{1/2}]^{1/2}+(R^2-Y^2)^{1/2}+a}{[R^2+a^2-2a(R^2-Y^2)^{1/2}]^{1/2}-(R^2-Y^2)^{1/2}+a}\right) \mathrm{d}Y (a > R) \\ 2G\rho \displaystyle\int_0^R \ln\left(\frac{[L^2+2R^2+2R(R^2-Y^2)^{1/2}]^{1/2}-(R^2-Y^2)^{1/2}-R}{[L^2+2R^2-2R(R^2-Y^2)^{1/2}]^{1/2}+(R^2-Y^2)^{1/2}-R}\right) \mathrm{d}Y \\ \quad +4G\rho R \qquad (a = R) \\ 2G\rho \displaystyle\int_0^R \ln\left(\frac{[L^2+R^2+a^2+2a(R^2-Y^2)^{1/2}]^{1/2}-(R^2-Y^2)^{1/2}-a}{[L^2+R^2+a^2-2a(R^2-Y^2)^{1/2}]^{1/2}+(R^2-Y^2)^{1/2}-a}\right) \mathrm{d}Y \\ \quad +2G\rho \displaystyle\int_0^R \ln([R^2+a^2+2a(R^2-Y^2)^{1/2}]^{1/2}+(R^2-Y^2)^{1/2}+a)\,\mathrm{d}Y \\ \quad +2G\rho \displaystyle\int_0^R \ln([R^2+a^2-2a(R^2-Y^2)^{1/2}]^{1/2}+(R^2-Y^2)^{1/2}-a)\,\mathrm{d}Y \\ \quad -4G\rho R(\ln R - 1) \qquad (a < R). \end{cases} \tag{19}$$

Challenges of cylindrical simulations

Gravitational force of a **sphere**

$$\mathbf{F}(r) = \begin{cases} -\dfrac{GMm}{r^2}\hat{\mathbf{r}}, & r \geq R, \\[2ex] -\dfrac{GMm}{R^3} r\hat{\mathbf{r}}, & 0 \leq r \leq R. \end{cases}$$

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Additional problems

- Mixed boundary conditions
- Ewald summation is necessary, as the simulation otherwise collapses due to numerical effects
- Ewald summation necessitates the force calculation for a finite cylinder instead of an infinite
- Custom initial condition generation code is needed to create physically and numerically sensible ICs in a half-periodic, half-stereographic geometry

Challenges of cylindrical simulations

Gravitational force of a **sphere**

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- Ewald summation necessitates the force calculation for a finite cylinder instead of an infinite
- Custom initial condition generation code is needed to create physically and numerically sensible ICs in a half-periodic, half-stereographic geometry
- The effects of perturbation applied to a rotating setting is still unknown

Gravitational force of a **cylinder**

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Thank you for your attention!

National Research, Development and Innovation Office
Hungary

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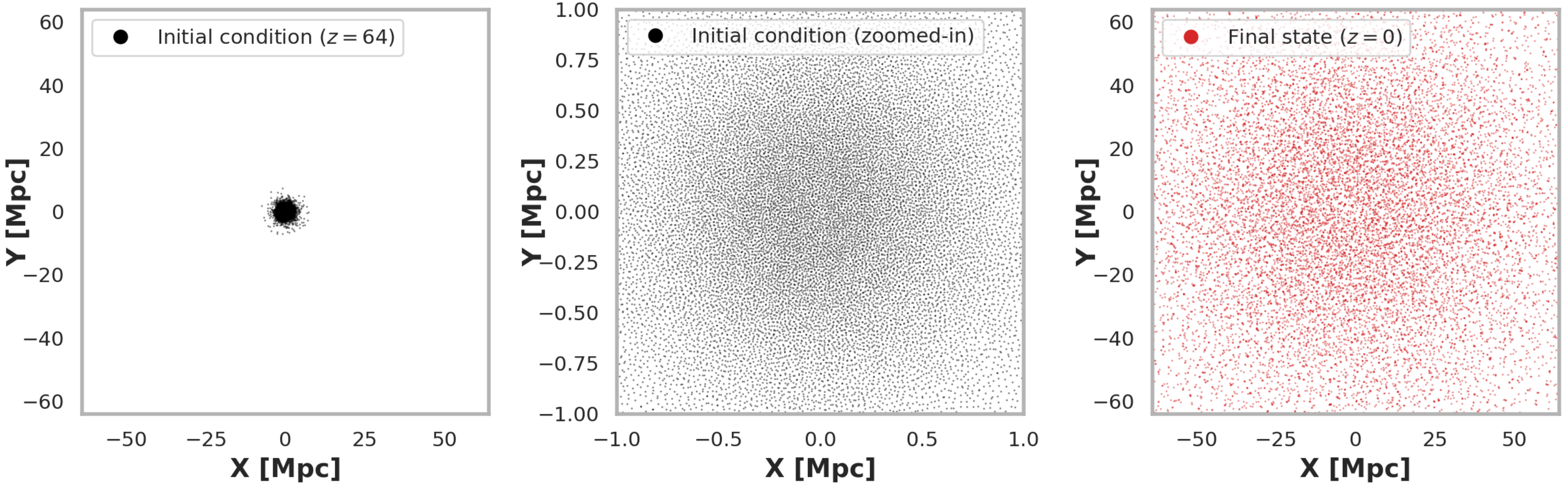
ELTE
Eötvös Loránd University

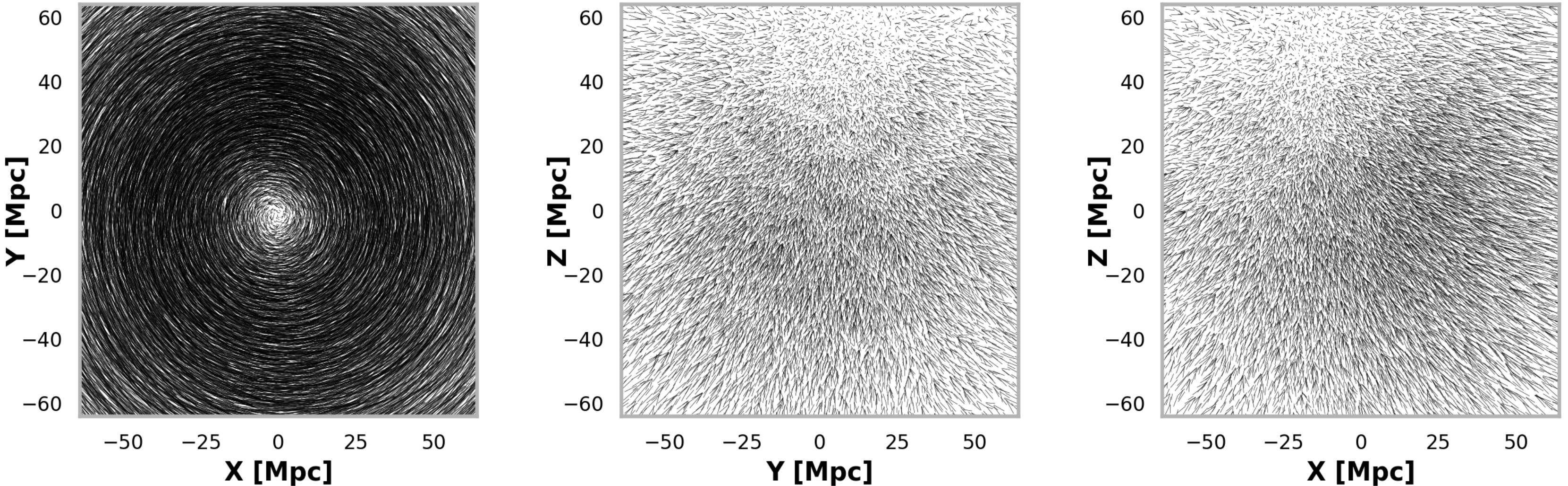
Extra Slides

StePS spherical rotating simulations

Expansion



Displacement field



Measuring the expansion rate in a rotating simulation