Microscopic structure of spacetime from **neutron star** oscillations

Anna Horváth
HUN-REN Wigner Research Centre for Physics
Eötvös Loránd University

In collaboration with:
Aneta Wojnar
Gergely Gábor Barnaföldi

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GPU Day 2026

The effects of strong gravity on the dispersion relation of massive particles in the Kaluza–Klein theory, (2025) arXiv: 2510.16631

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Extra dimensions?

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Insane environment...

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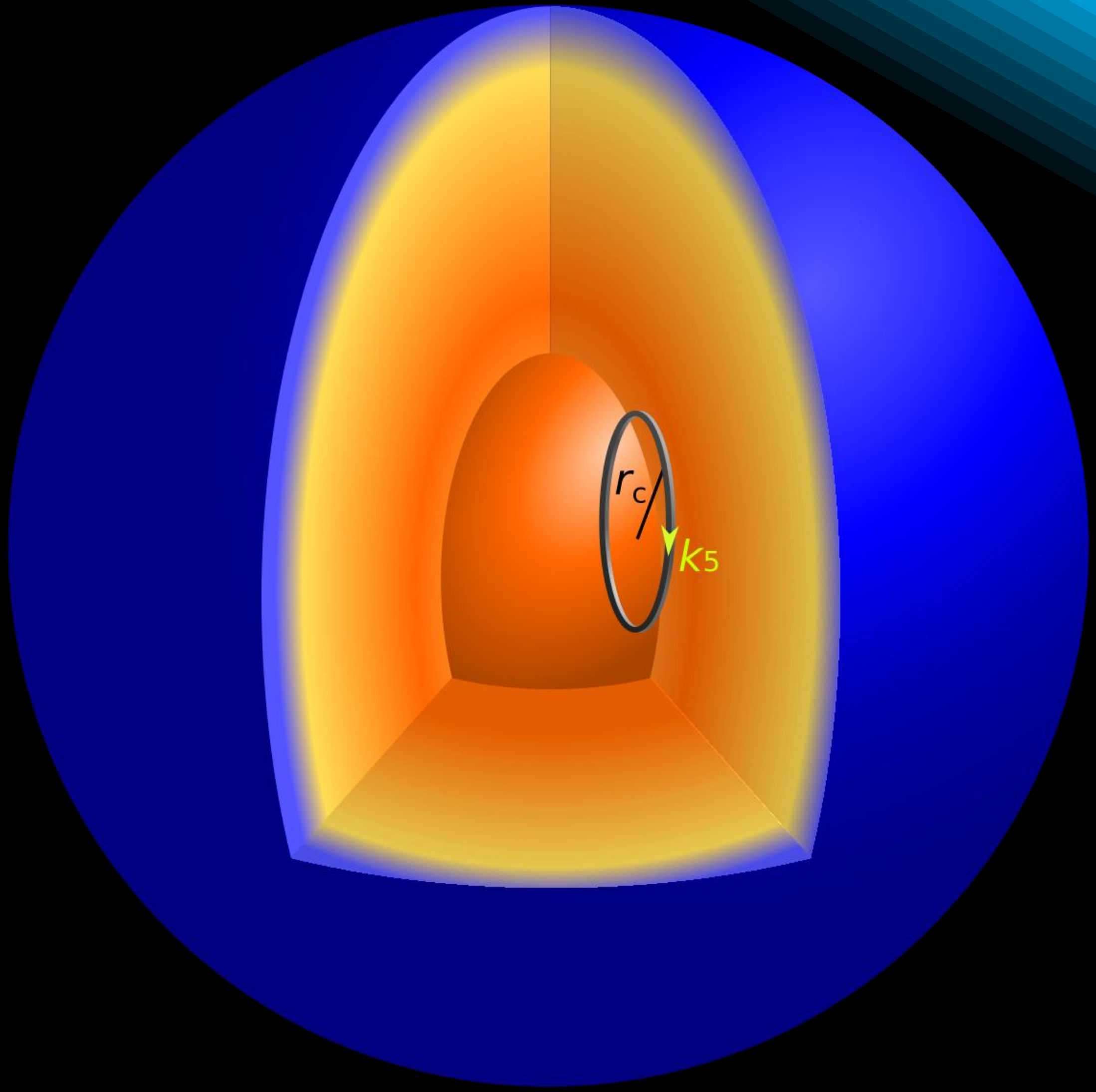
Modelling neutron stars

- Two ingredients are needed:



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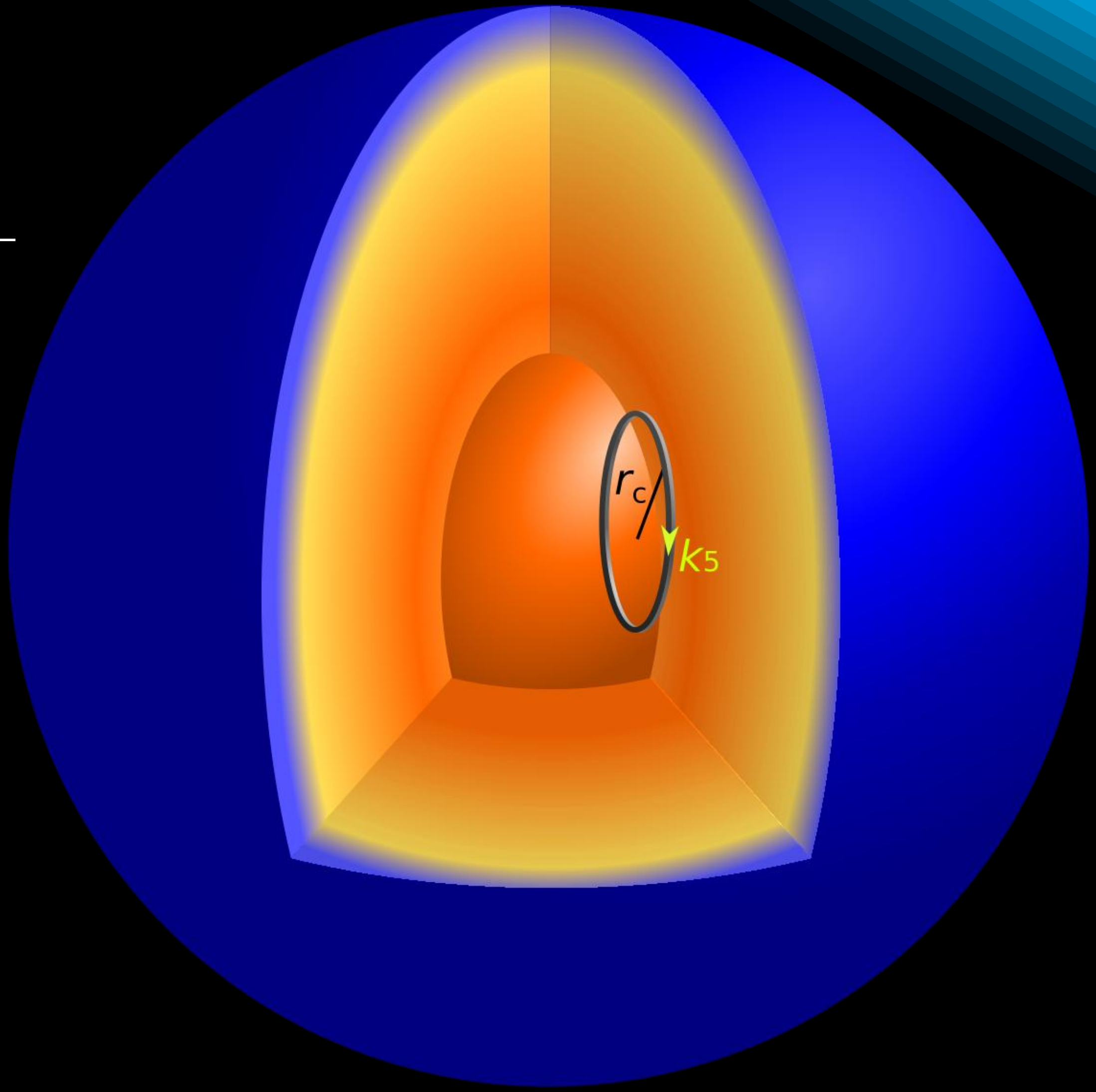

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Modelling neutron stars

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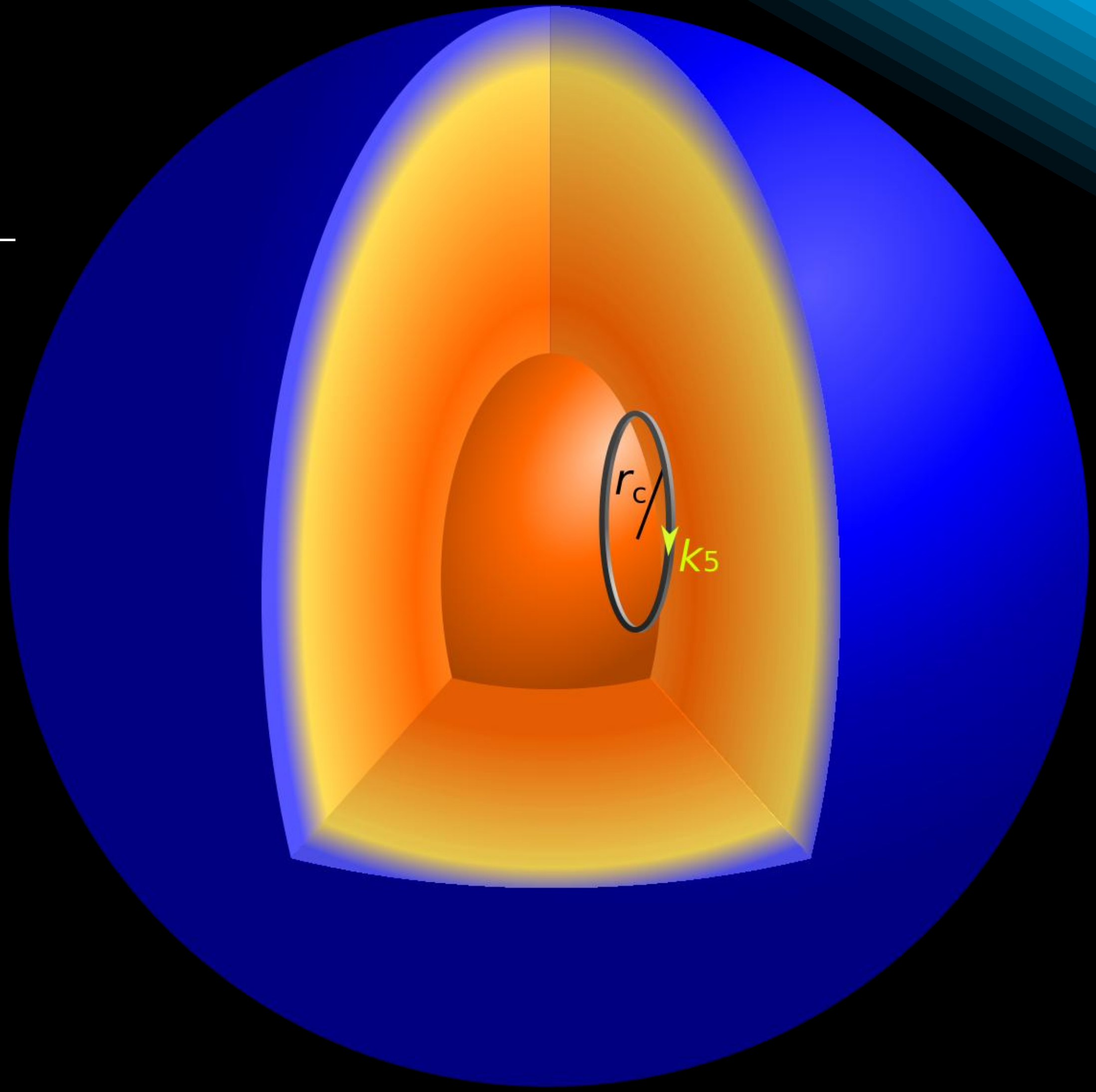

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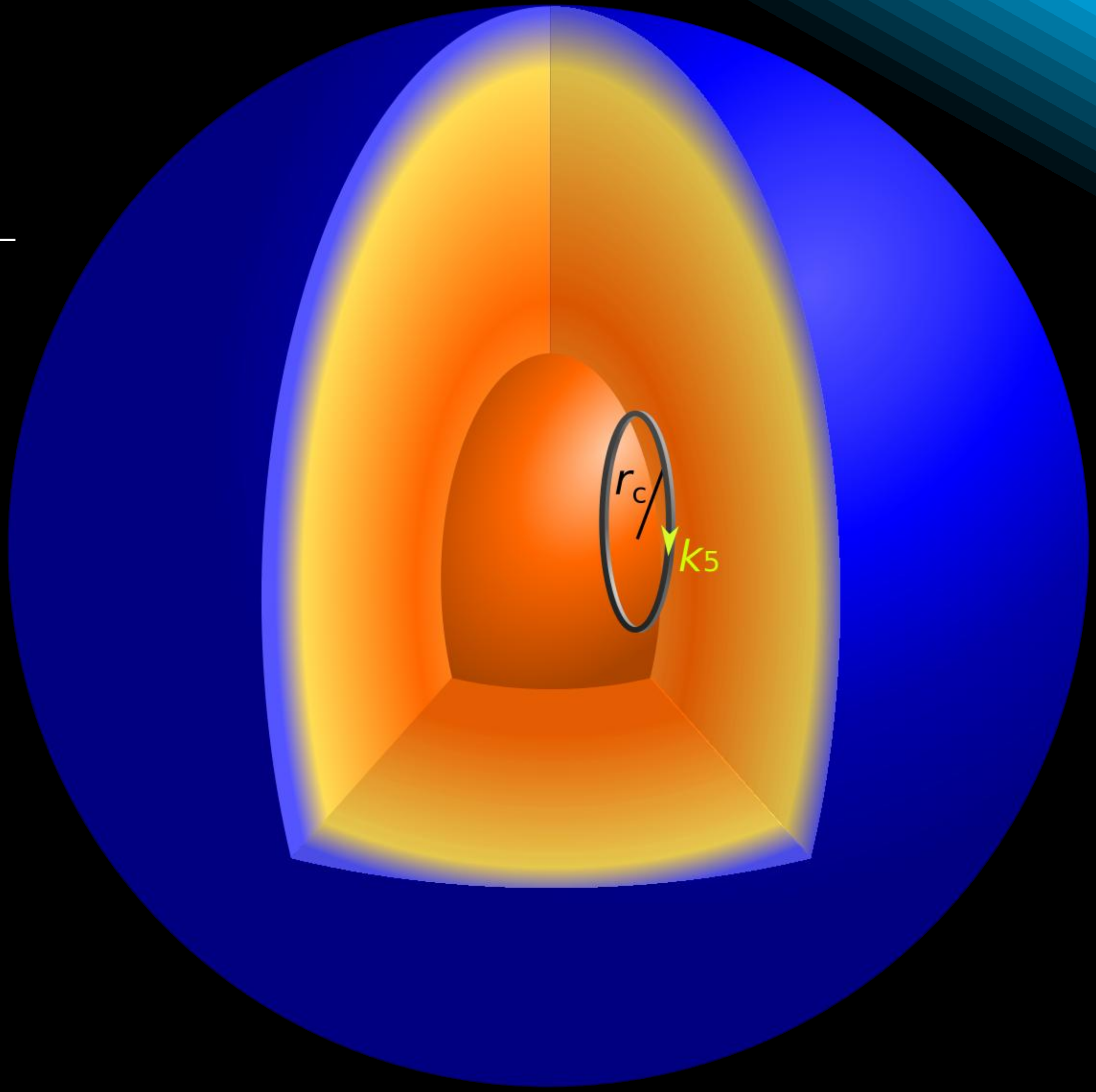

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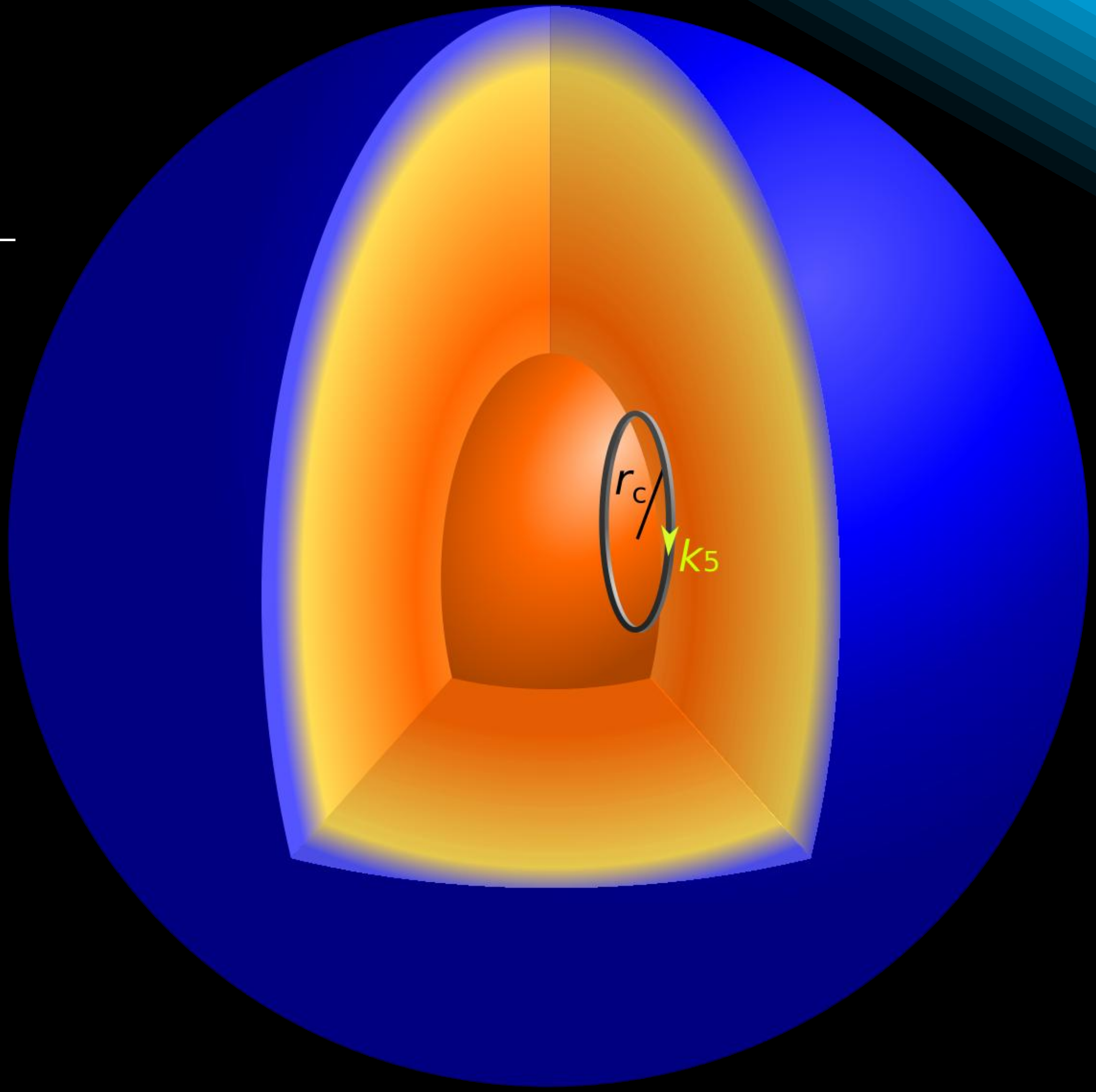

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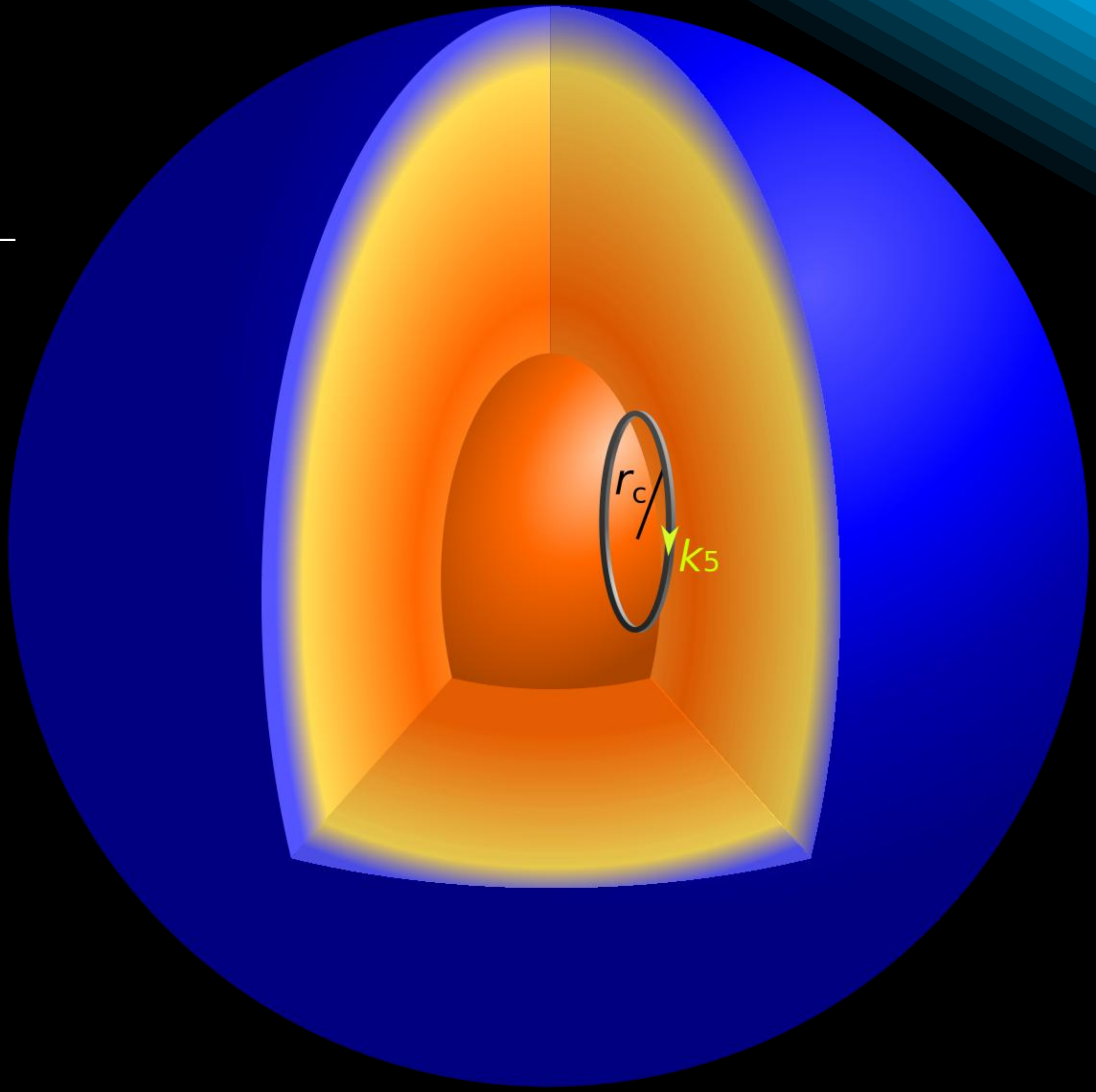
Hard!



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What I mean by extra dimensions:

$x =$ ( )

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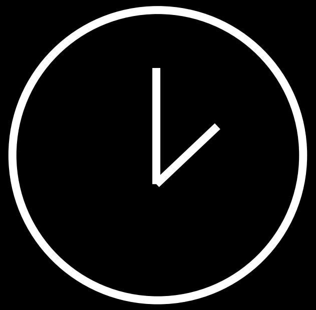

time

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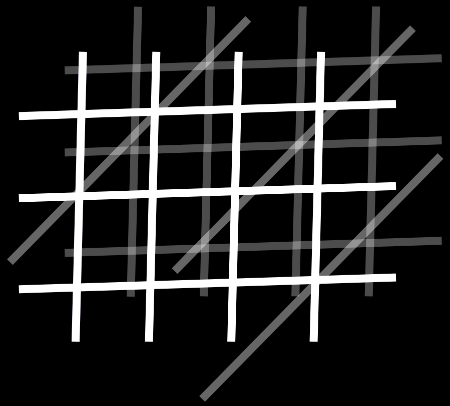
What I mean by extra dimensions:

$x = (t, \quad x_1, x_2, x_3, \quad)$



time **space**

spacetime

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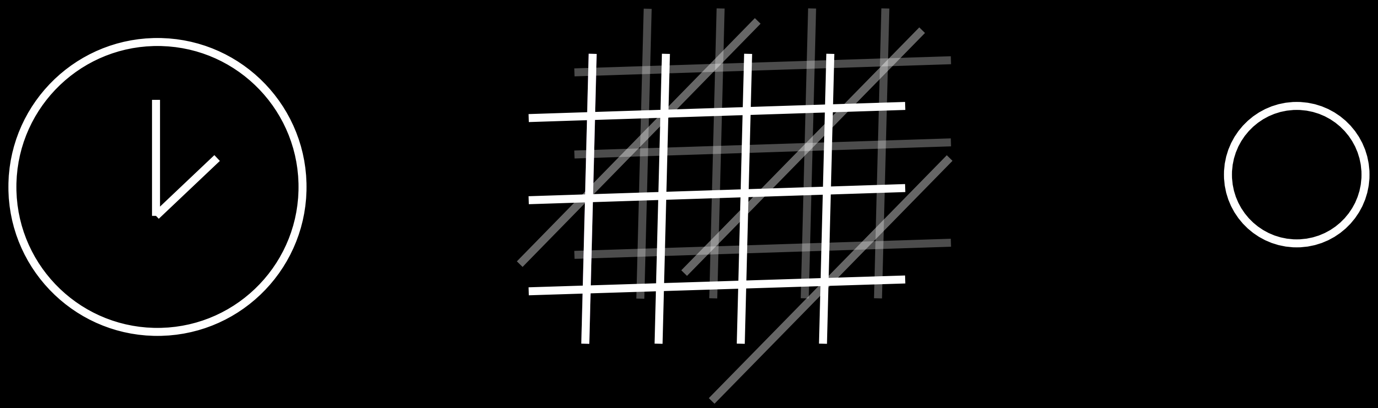
What I mean by extra dimensions:

$x = (t, x_1, x_2, x_3, x_5)$



time **space** **extra dimension**

spacetime

5D KK spacetime

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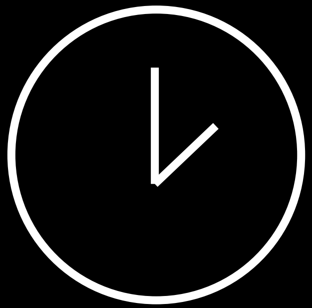

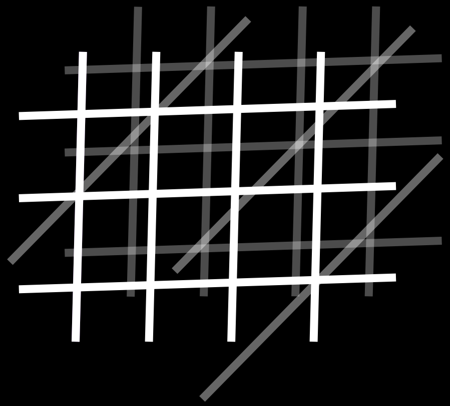

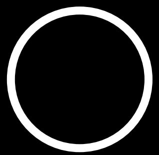

Spatial dimension

Compactified: curled up into a microscopic **circle**

time **space** **extra dimension**

Too small to see

spacetime

5D KK spacetime

How does this appear in the equations?

Let's see two possibilities!

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Particles move in the extra dimension

- Modification to the **equation of state**

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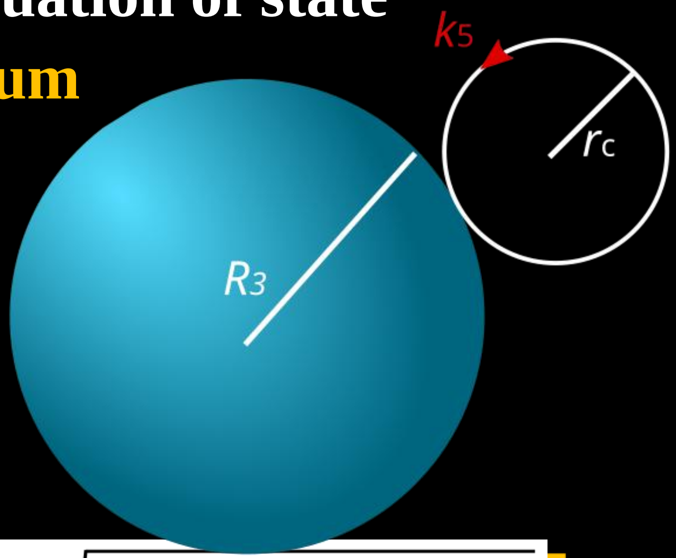

$$E = \sqrt{\mathbf{k}^2 + k_5^2 + m^2} = \sqrt{\mathbf{k}^2 + \left(\frac{N_{\text{exc}}}{r_{\text{c}}}\right)^2 + m^2}$$

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5^{th} dim. appears as a scalar field

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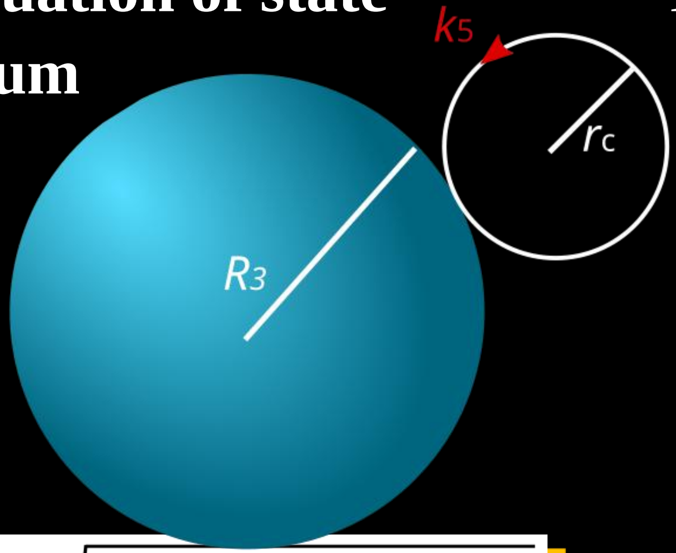

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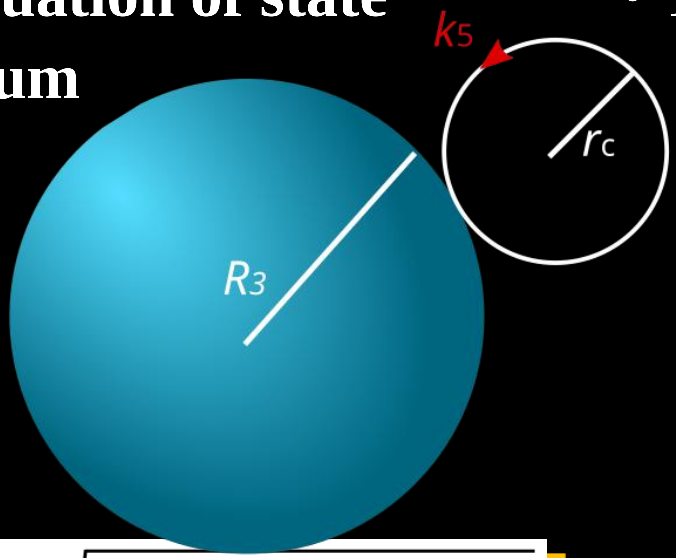

Is that all?

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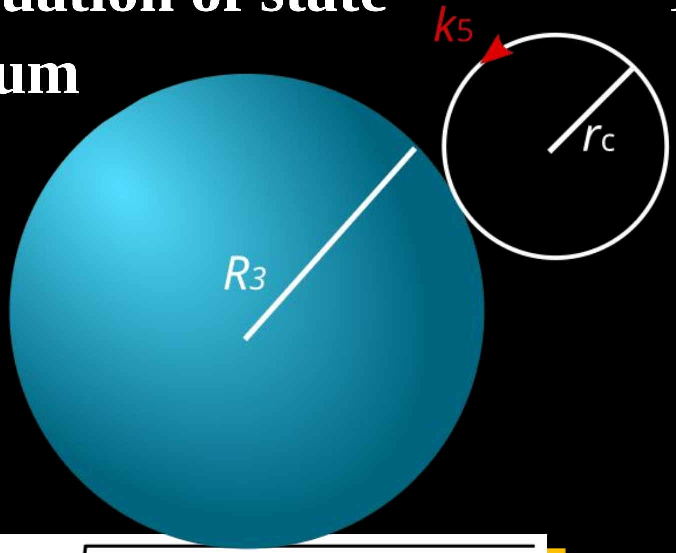

Is that all?

NO.

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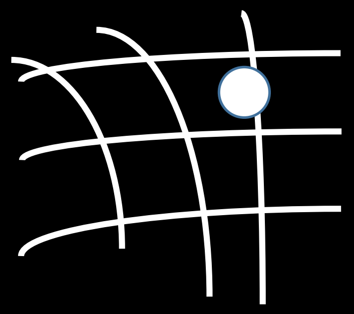
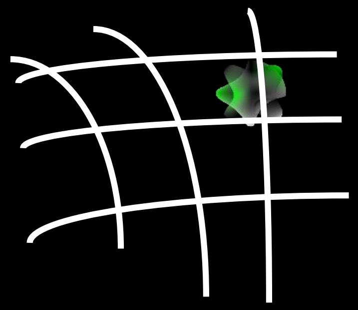
Particle on curved spacetime



Particle on curved spacetime

Classical picture

Particle on curved spacetime



Classical picture

We need quantum!

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Modifying Heisenberg’s uncertainty principle

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Modifying Heisenberg's uncertainty principle

- **Generalized Uncertainty Principle (GUP)**

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Our main focus

Extended uncertainty principle (EUP)

- Petruzziello et al.:

$$\sigma_{\mathrm{p}}\rho \gtrsim \pi\hbar \left[1 - \frac{\rho^2 \mathcal{R}|_{x_0}}{12\pi^2} + \xi \frac{\rho^4}{\lambda_C^2} \bar{\nabla}_j N_i \bar{\nabla}^j N^i \bigg|_{x_0}\right]$$

L. Petruzziello, F. Wagner, PRD 103, 104061 (2021)

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- Schwarzschild metric of a static, spherically symmetric black hole

$$\mathrm{d}s^2=-\left(1-\frac{2M}{r}\right)\mathrm{d}t^2+\left(1-\frac{2M}{r}\right)^{-1}\mathrm{d}r^2+r^2\mathrm{d}\Omega^2$$

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- Schwarzschild metric of a static, spherically symmetric black hole

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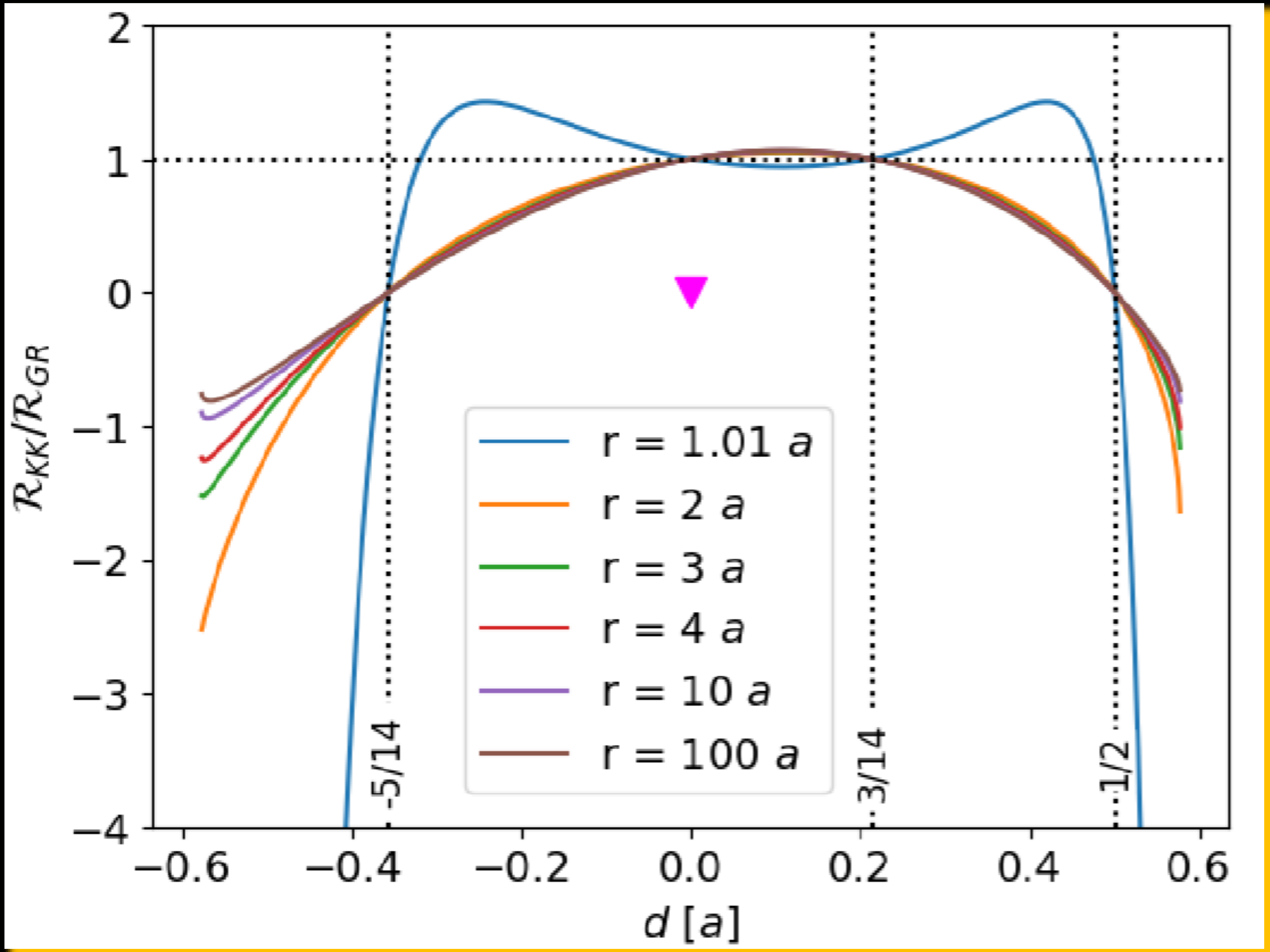
- Kaluza–Klein Schwarzschild metric

$$\mathrm{d}s^2 = -\left(1 - \frac{a}{r}\right)^{\frac{b-d}{a}}\mathrm{d}t^2 + \left(1 - \frac{a}{r}\right)^{-\frac{b+d}{a}}\mathrm{d}r^2 + r^2\left(1 - \frac{a}{r}\right)^{1-\frac{b+d}{a}}\mathrm{d}\Omega^2$$

R. Coquereaux, G. Esposito-Farese, in Annales de l’IHP Physique Théorique, Vol. 52 (1990) pp. 113-150.

General Relativity vs. Kaluza–Klein



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Modified Dispersion Relation

$$p^\mu p_\mu = \overbrace{k^\mu k_\mu}^{-m^2} - \frac{\hbar^2 \mathcal{R}}{6} + \hbar^2 \xi \frac{\rho^2}{2\lambda\!\!\!^{-}{}_{\mathrm{C}}^2} \bar{\nabla}_j N_i \bar{\nabla}^j N^i = -m_{\mathrm{eff}}^2$$

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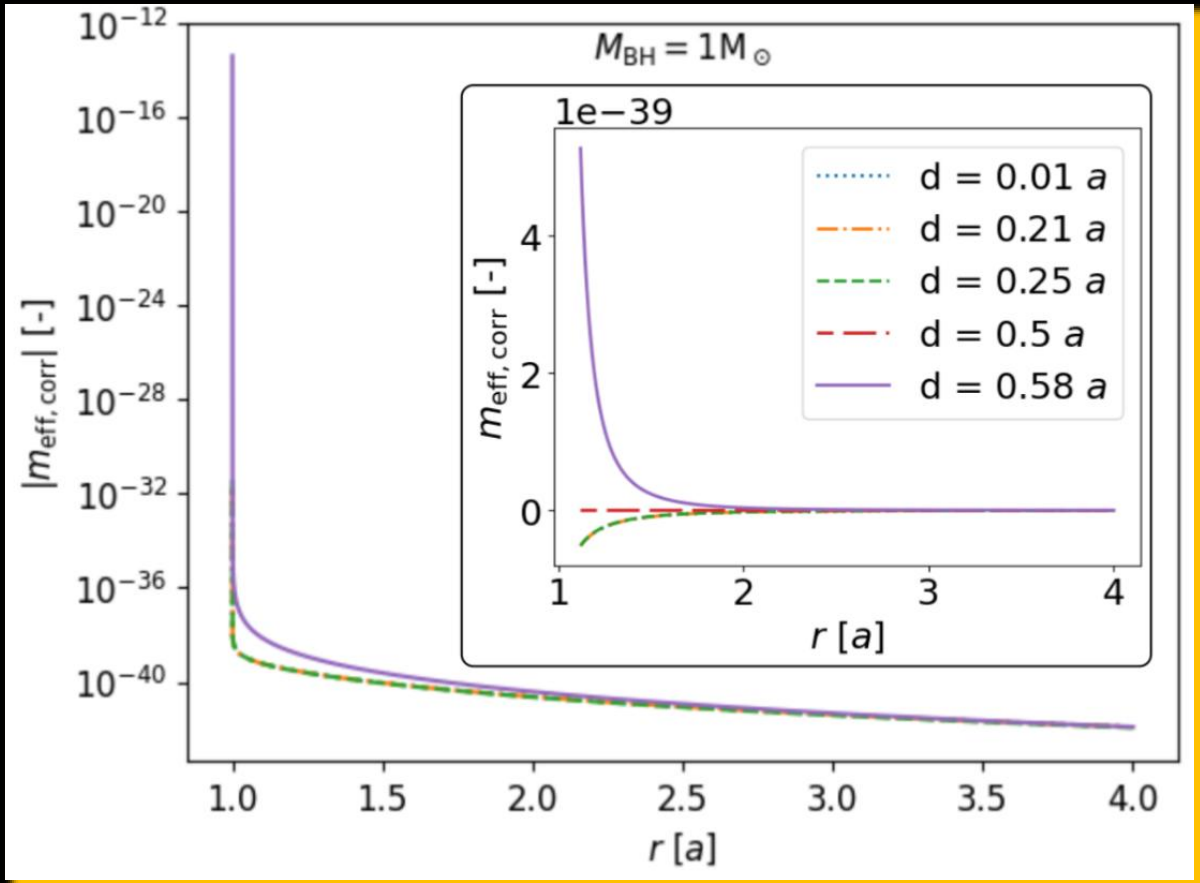

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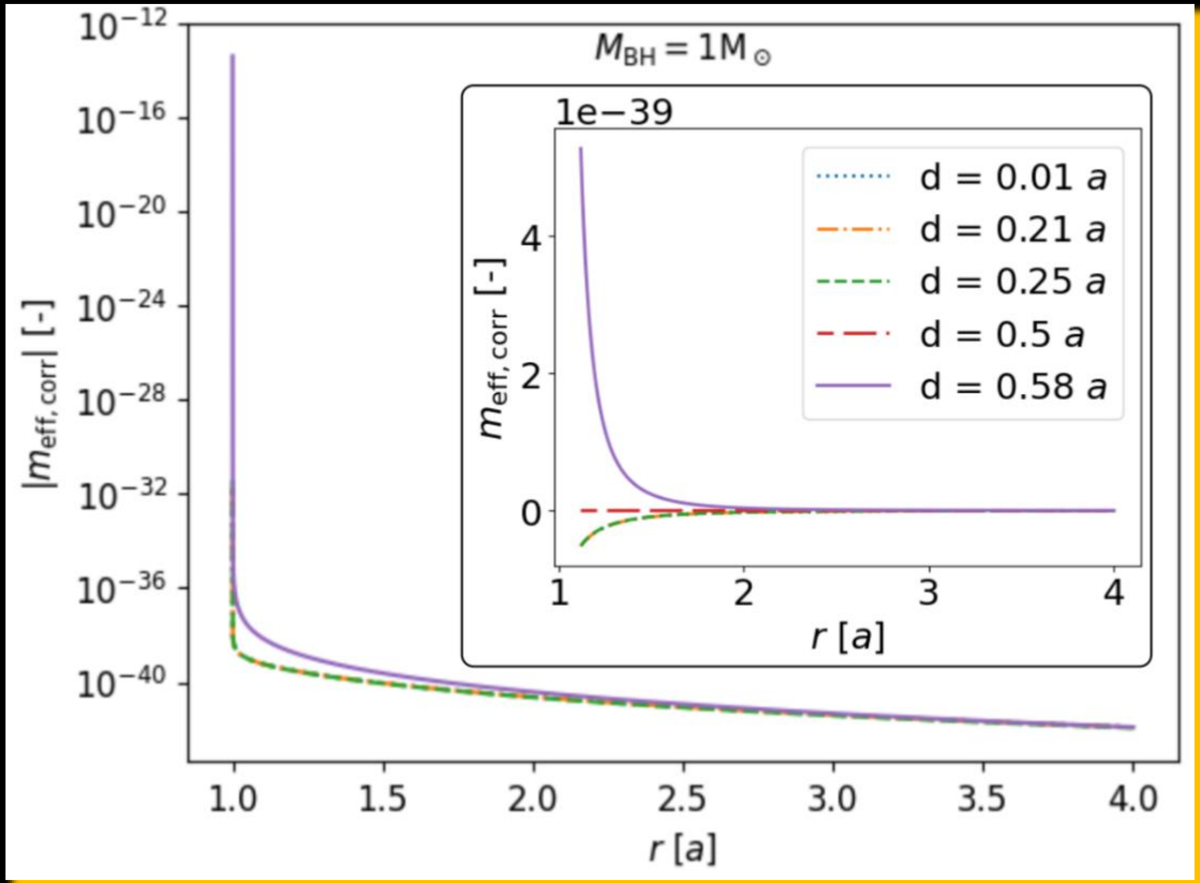

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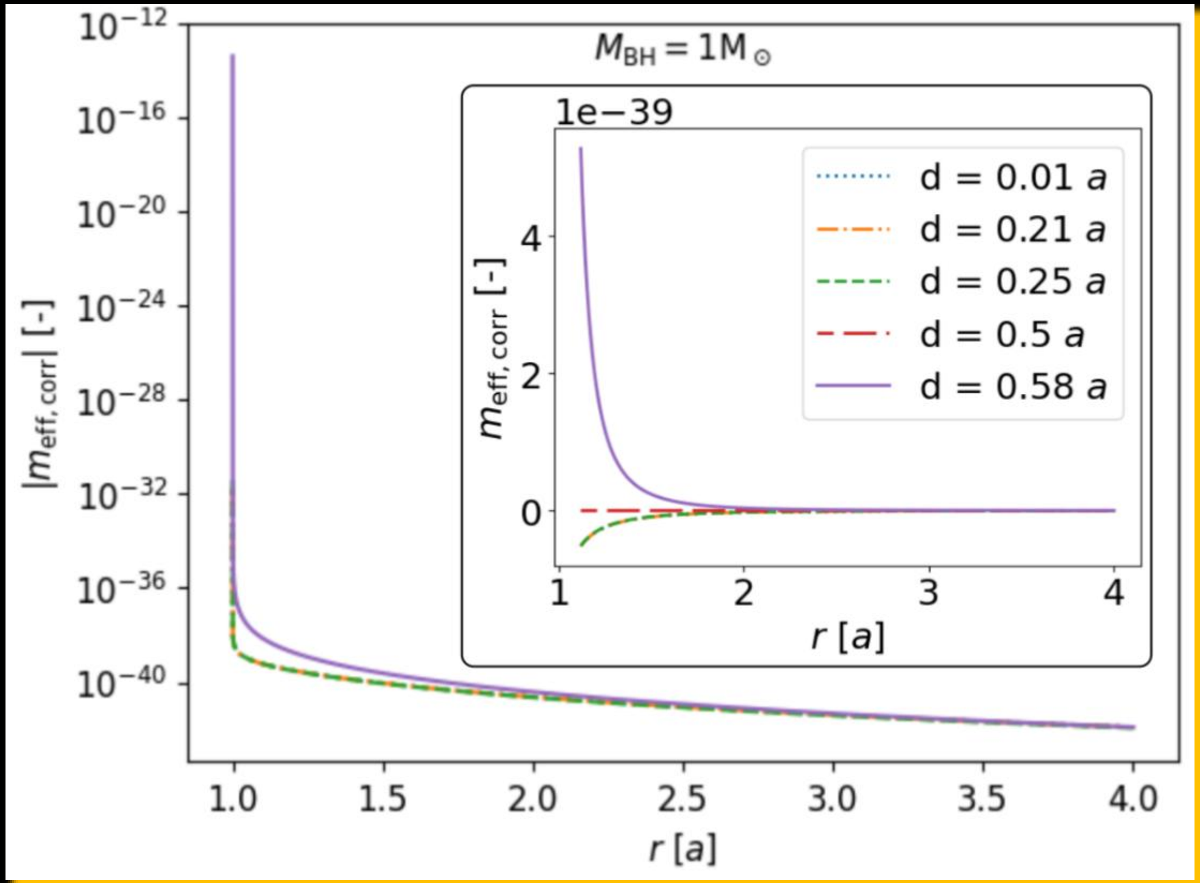
- Particle decay? Hawking radiation?



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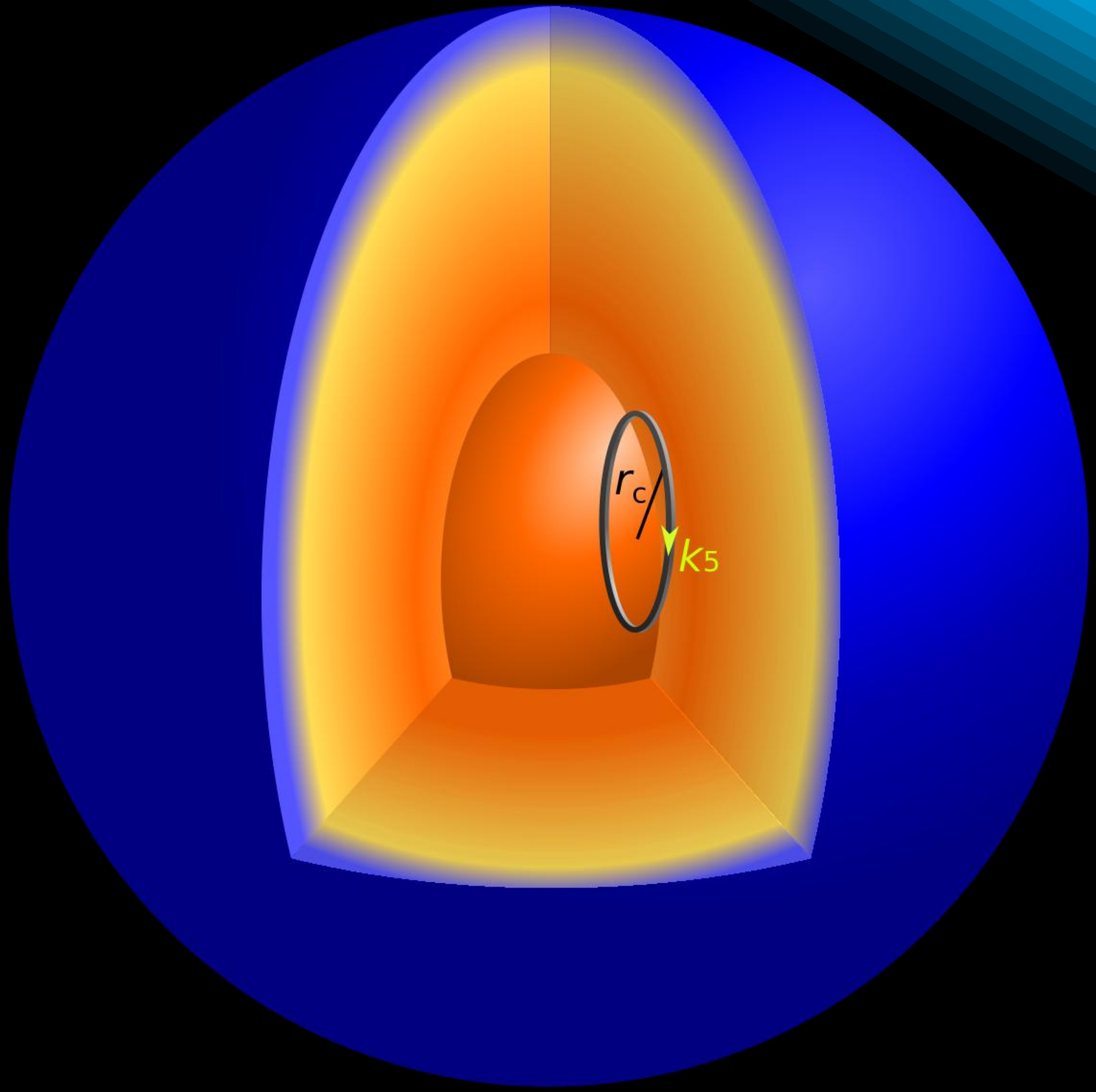
What can we see in neutron stars?

- Equation of state



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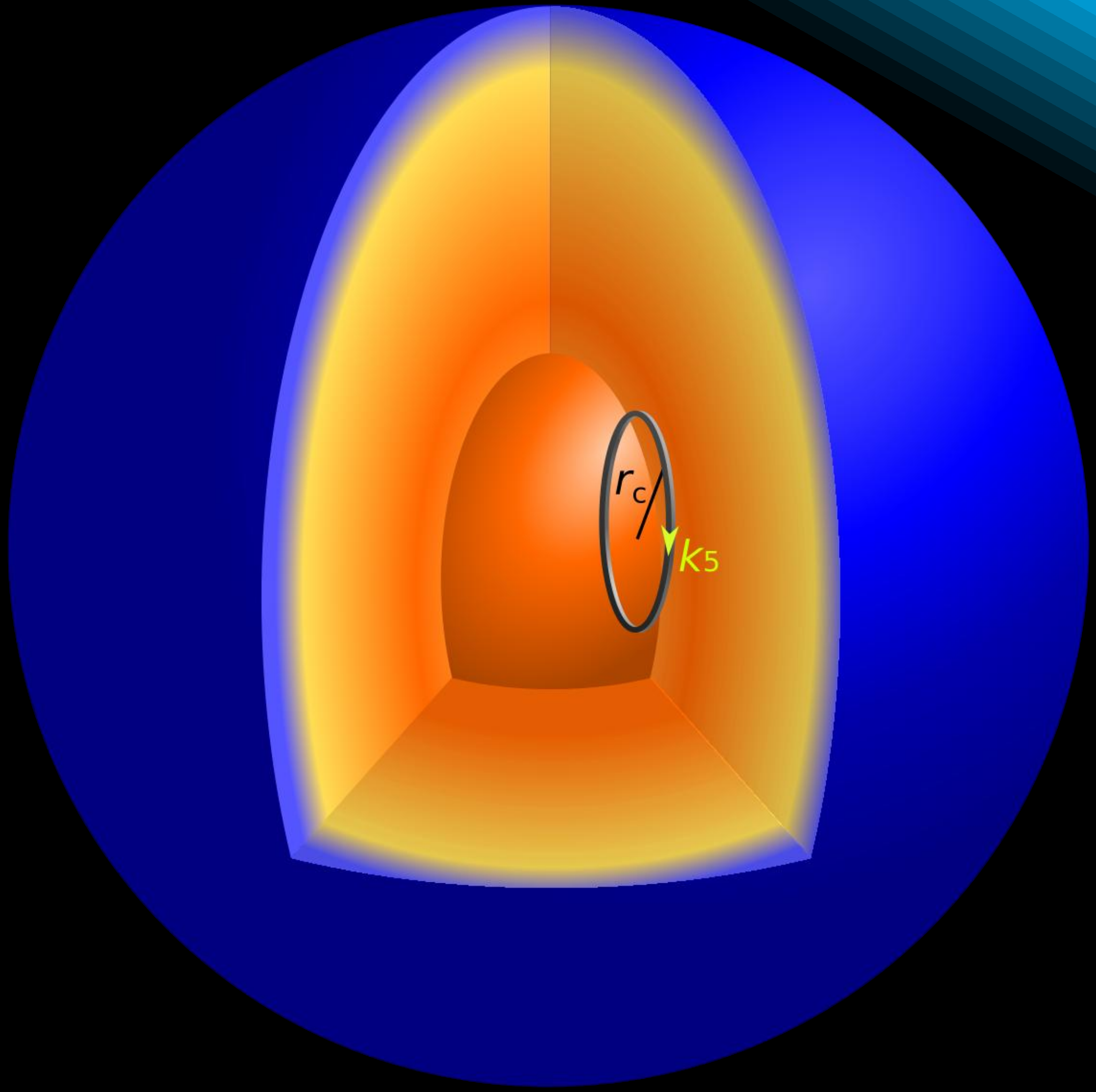

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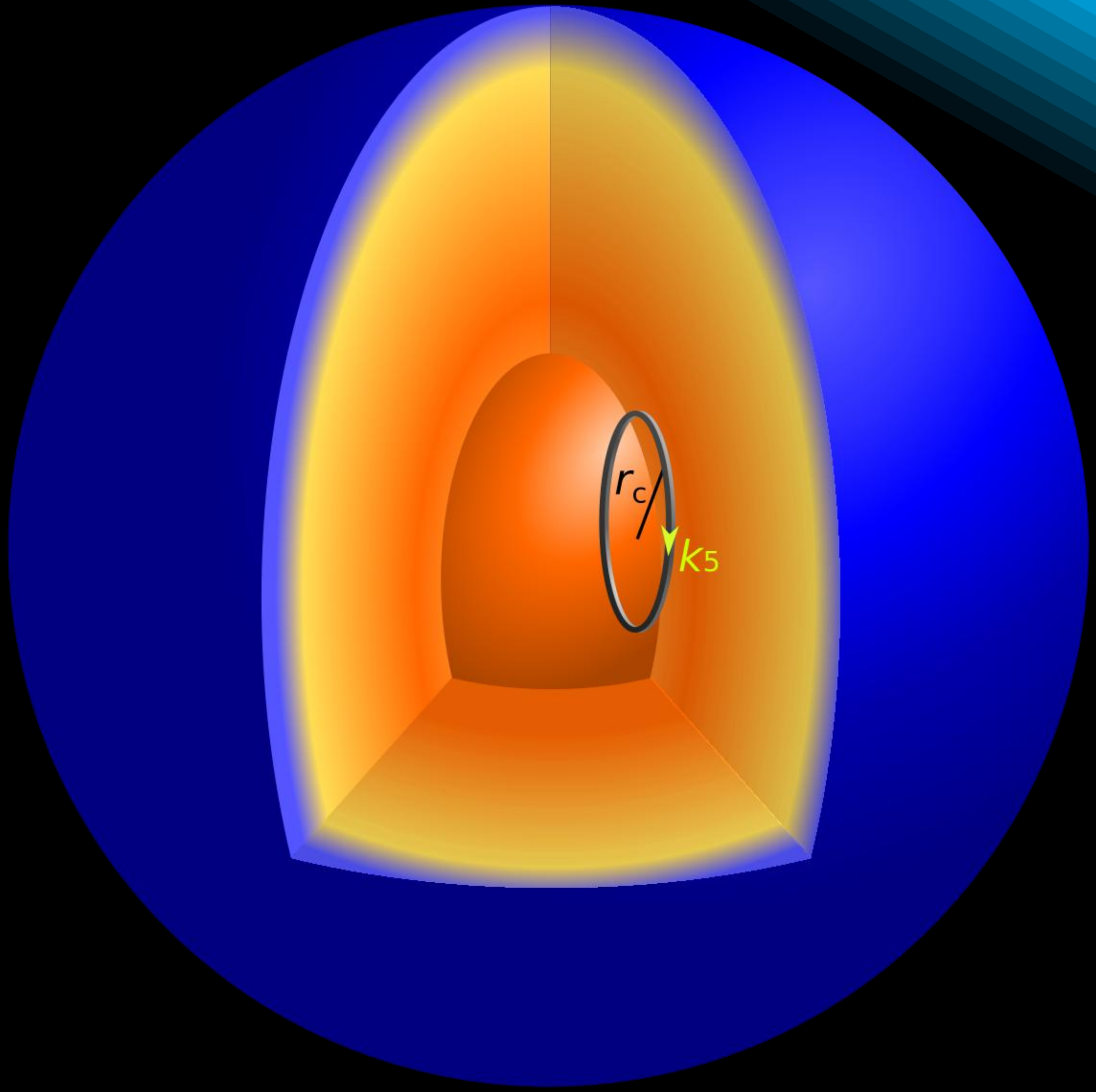

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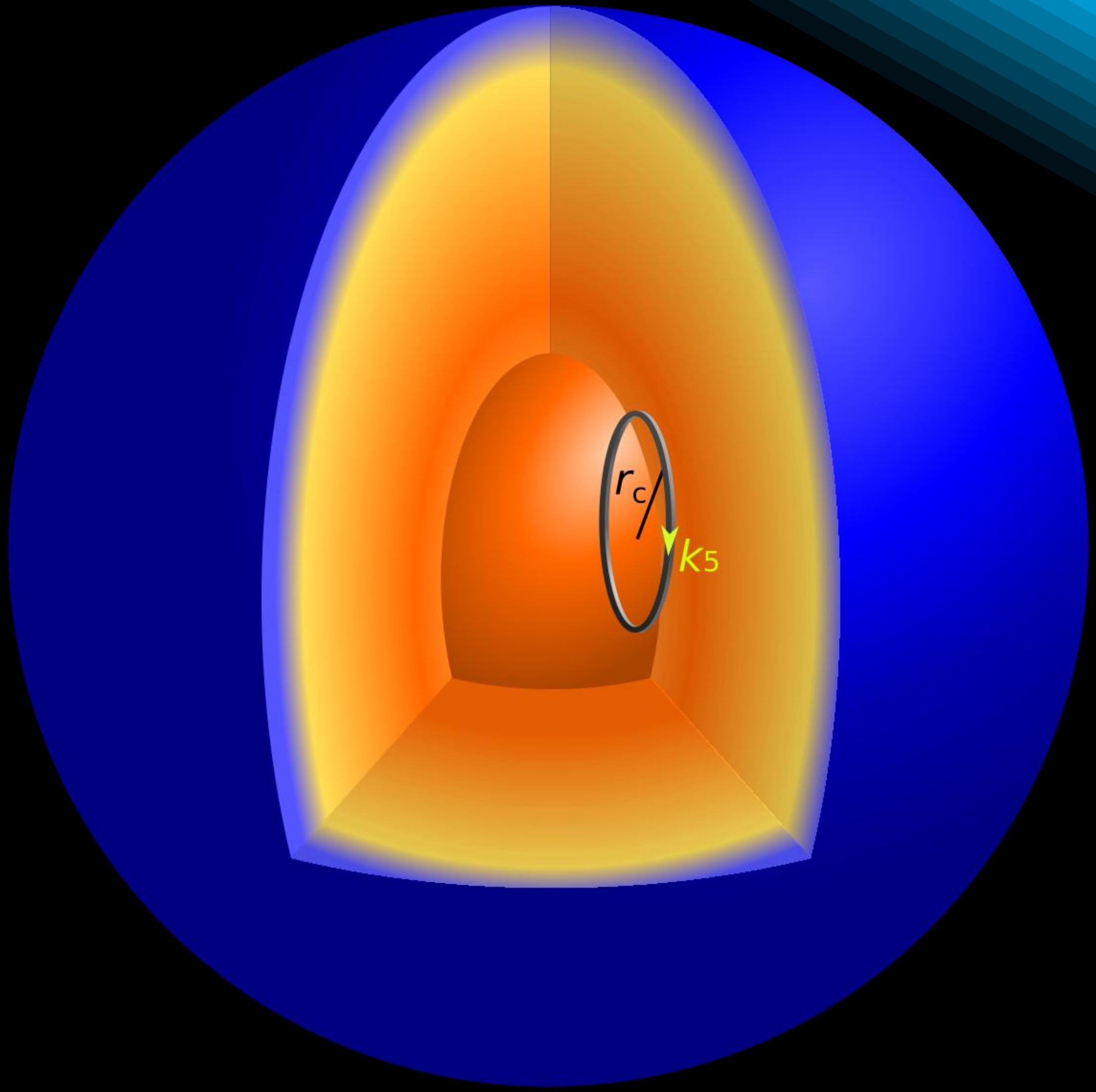
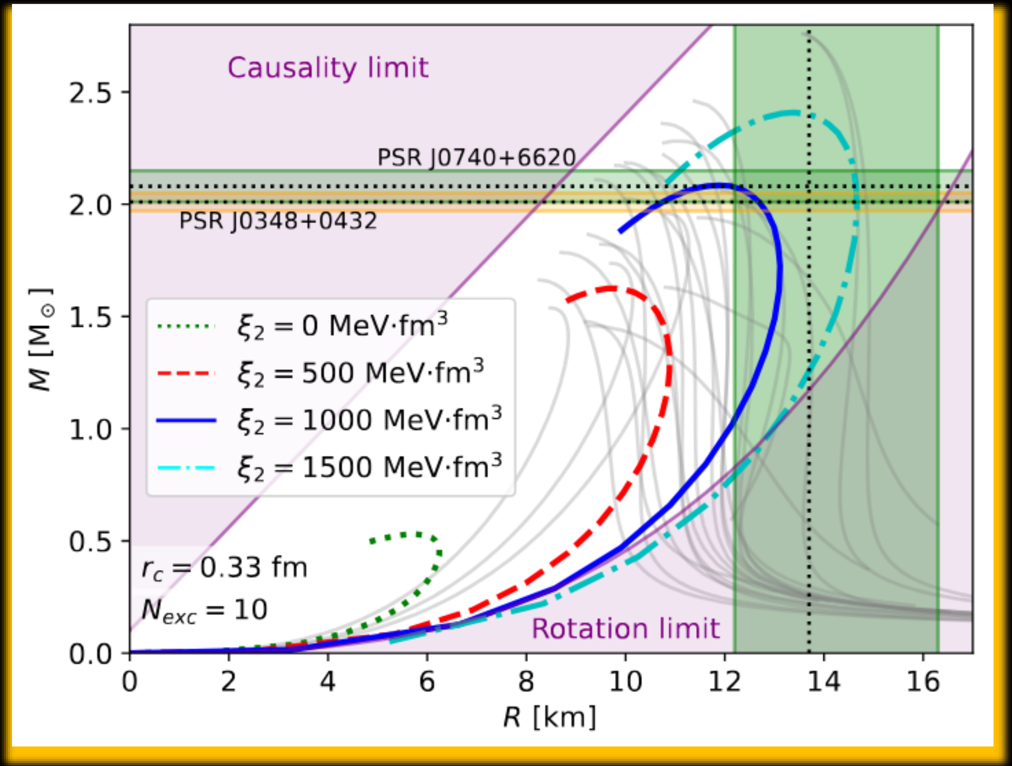
- **Numerical modelling**
  - Isolated neutron star
  - Static, spherically symmetric
  - Mass – radius curves
  - Maximal mass



Mass – radius relation

Previously: A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi, MNRAS 536 (2024) 1, 816-826



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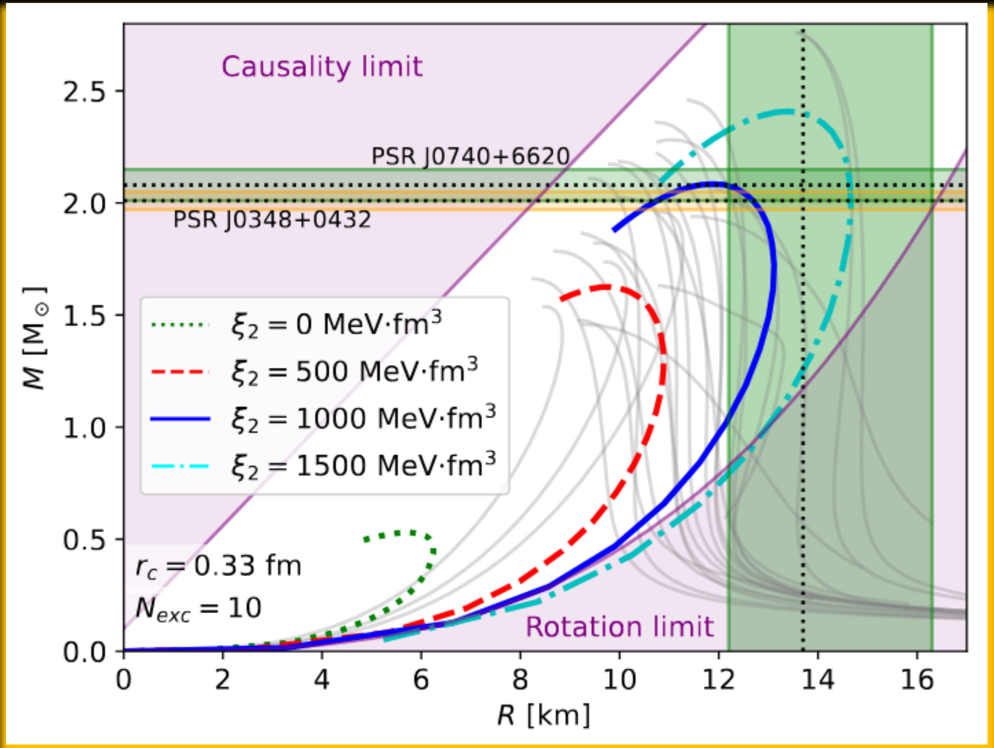

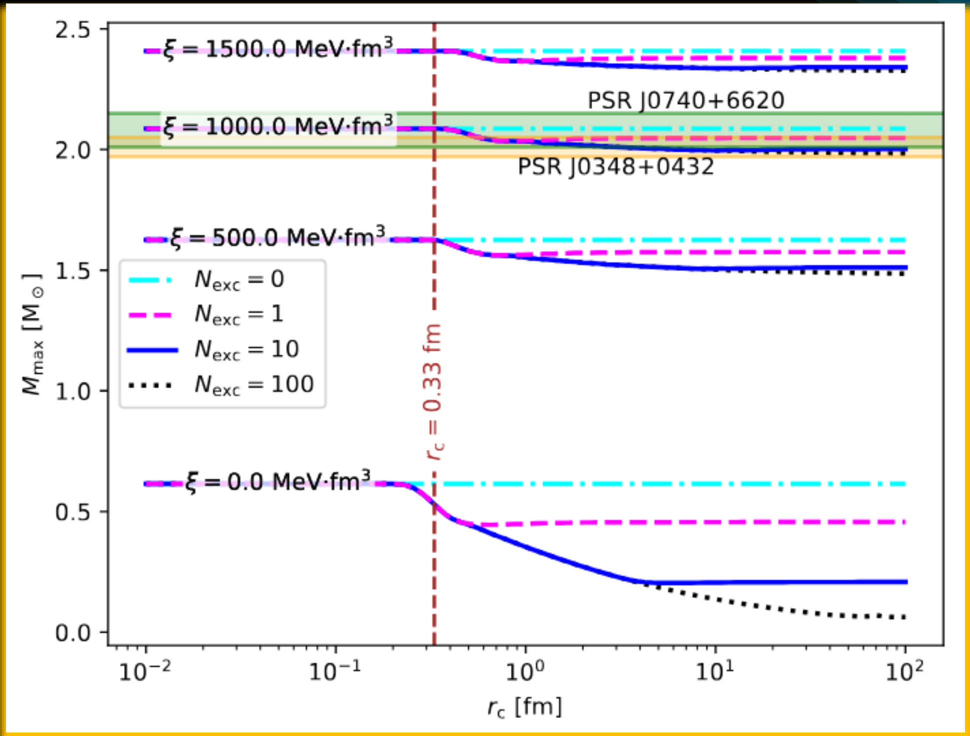

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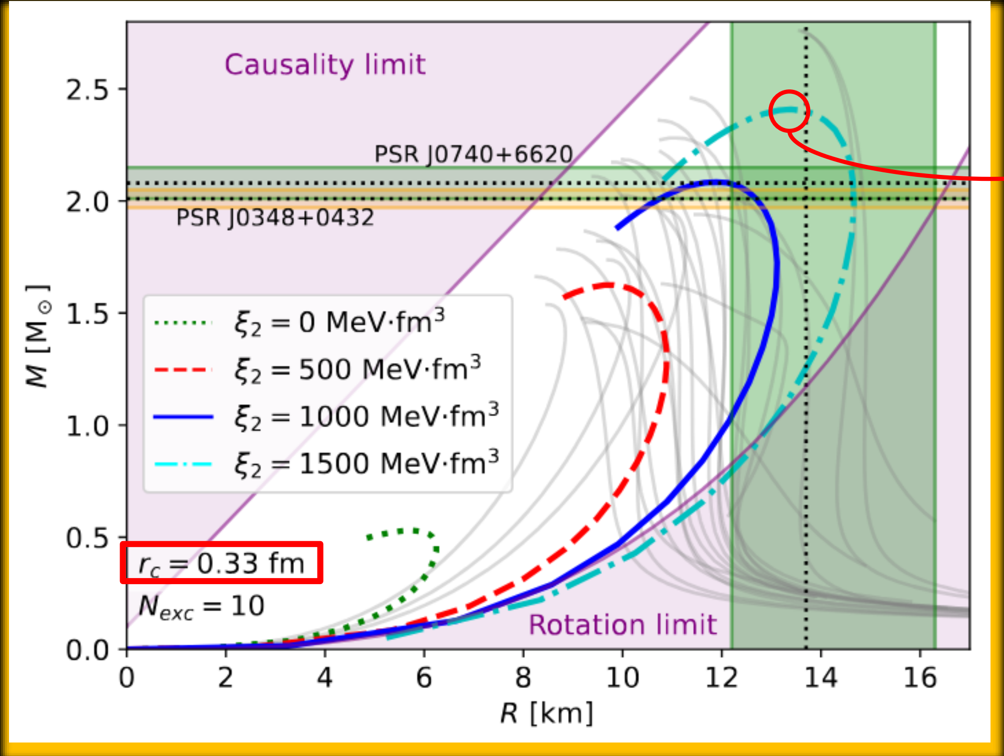

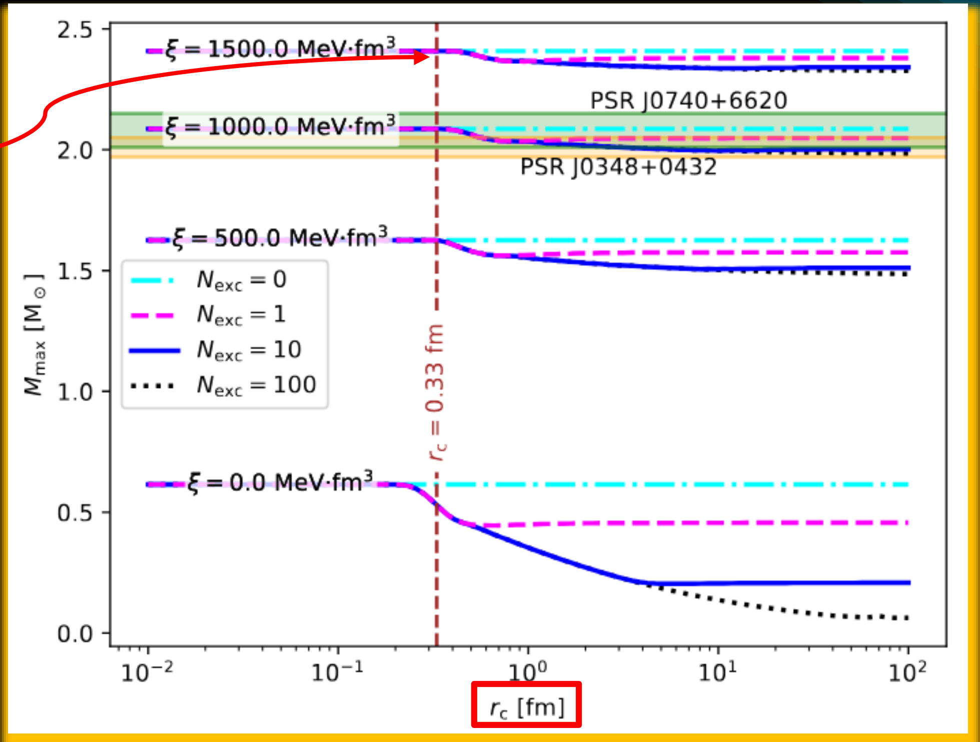

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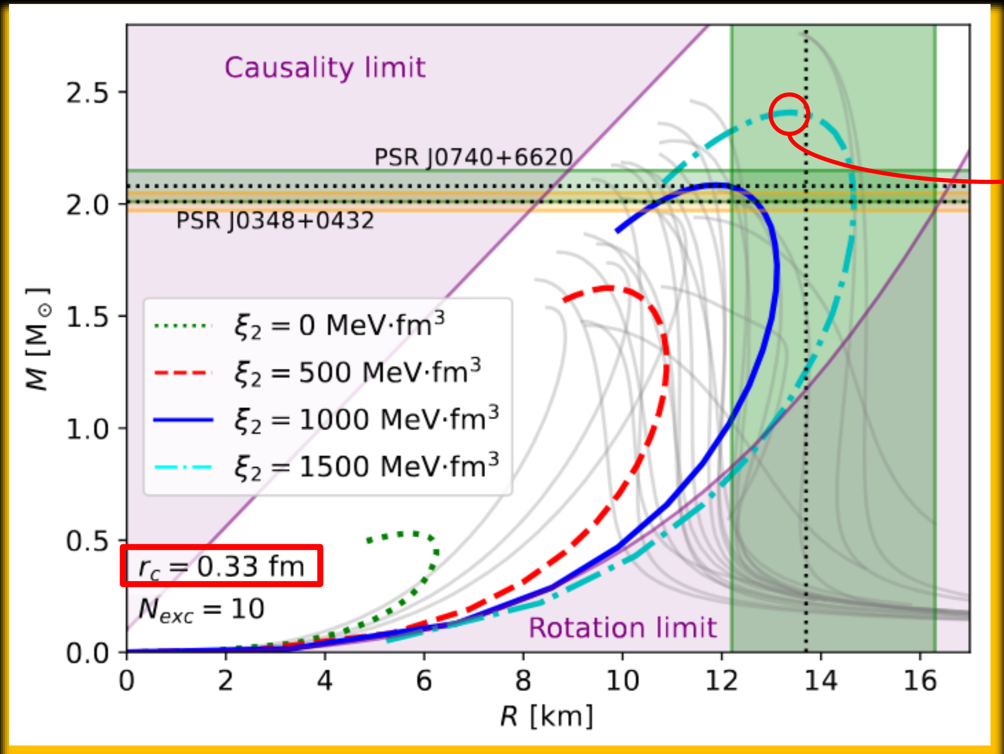

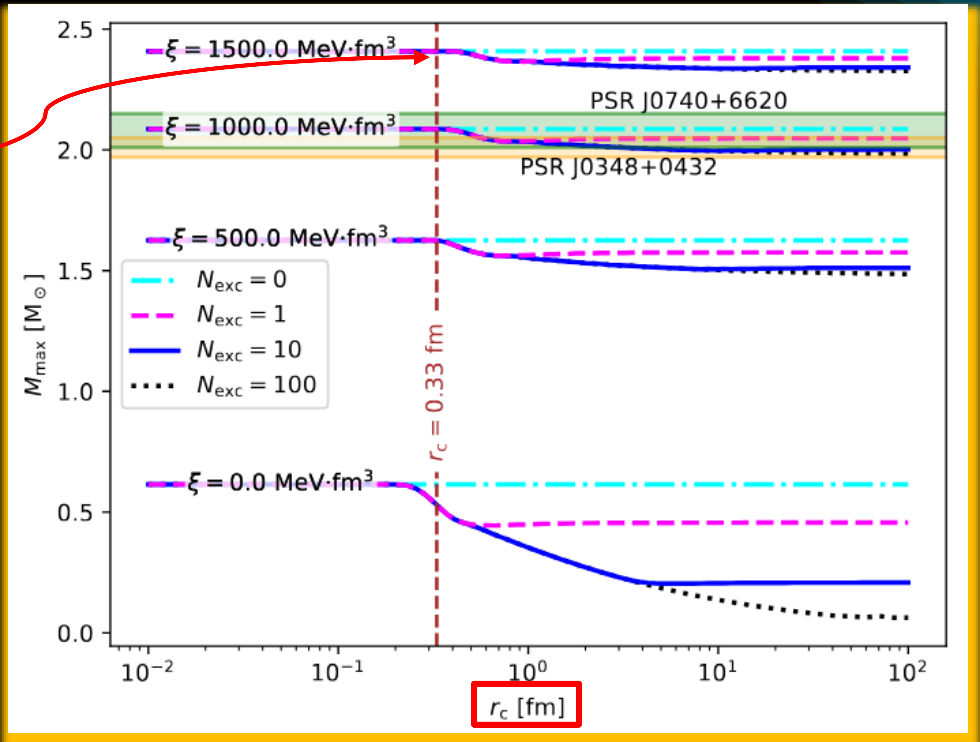

MANY EoSs satisfy data!

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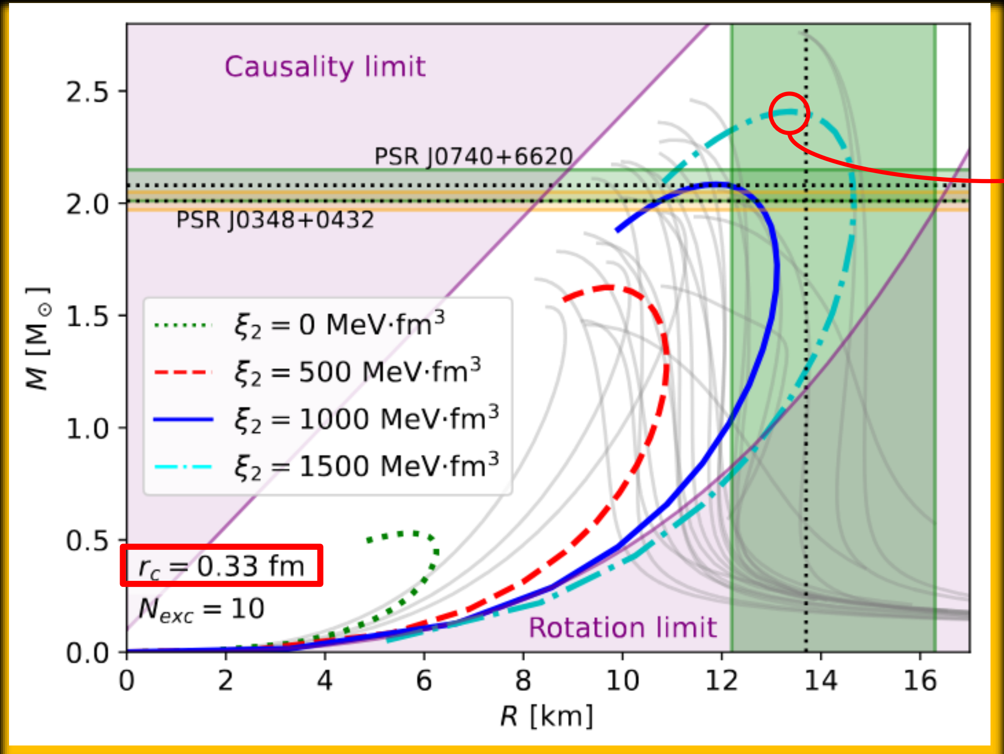

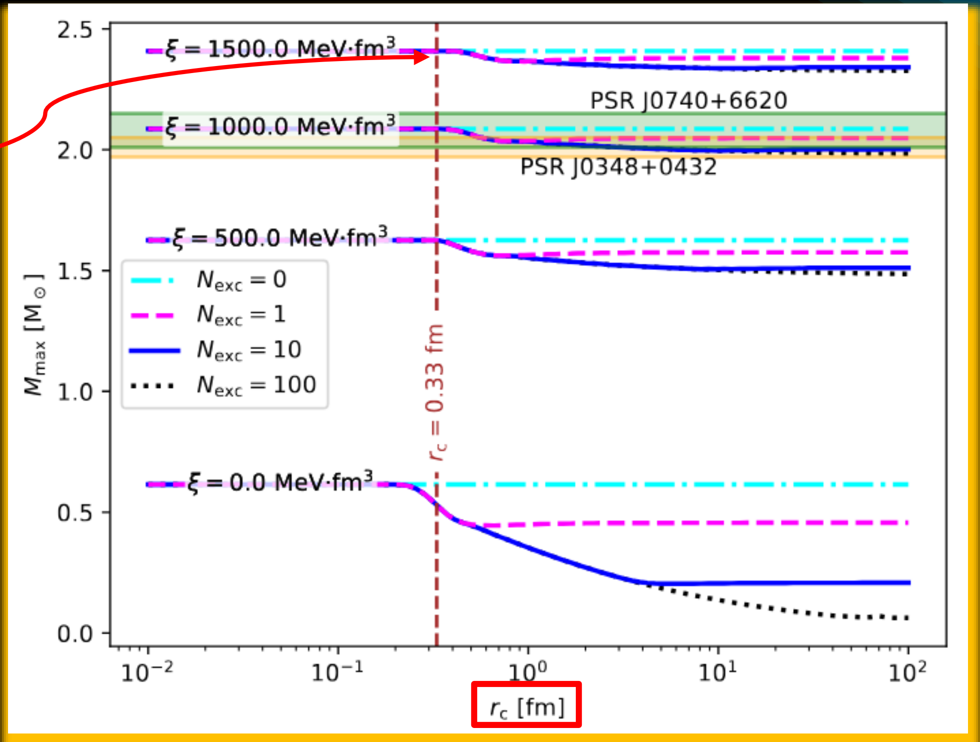

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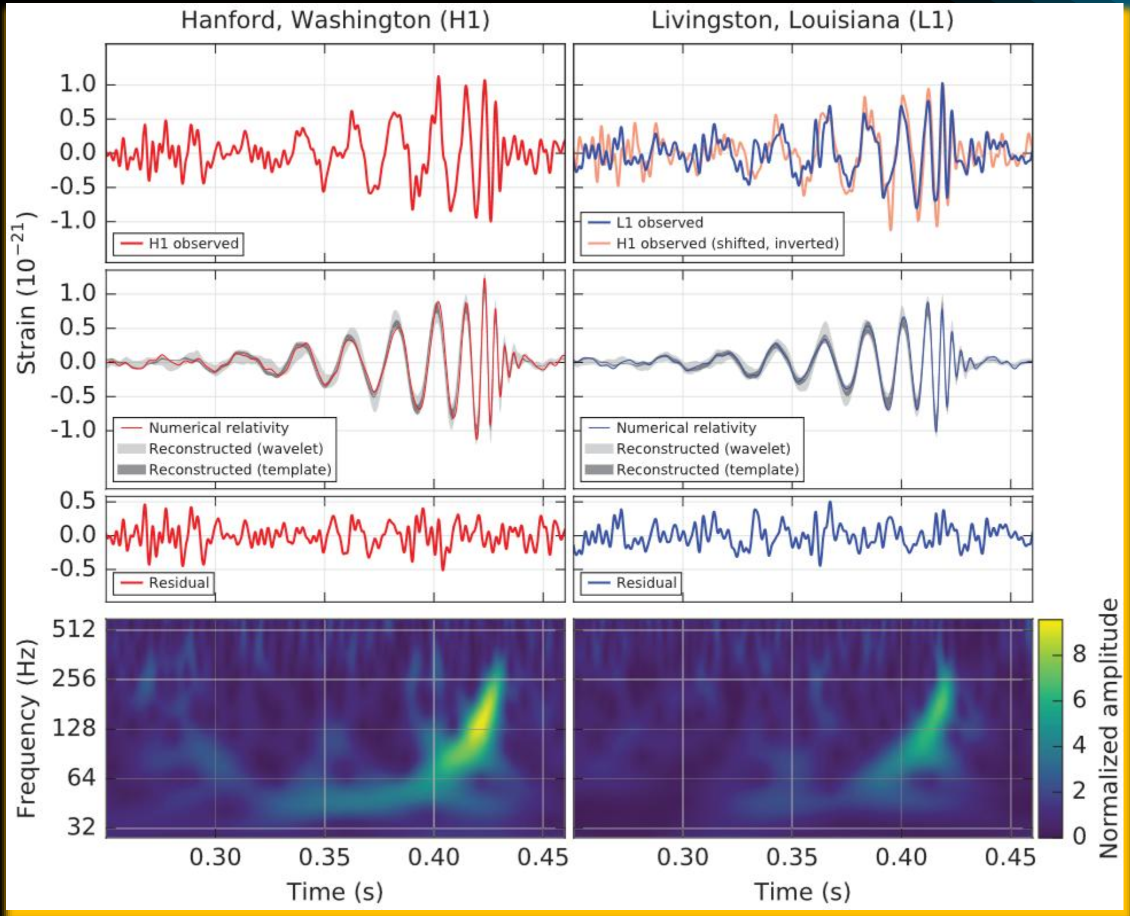
What to do?

Dynamical scenarios

- **Asteroseismology**
    - Continuous gravitational waves
    - Merger
    - Oscillation after events
    - Quasi-Periodic Oscillations (X-ray)



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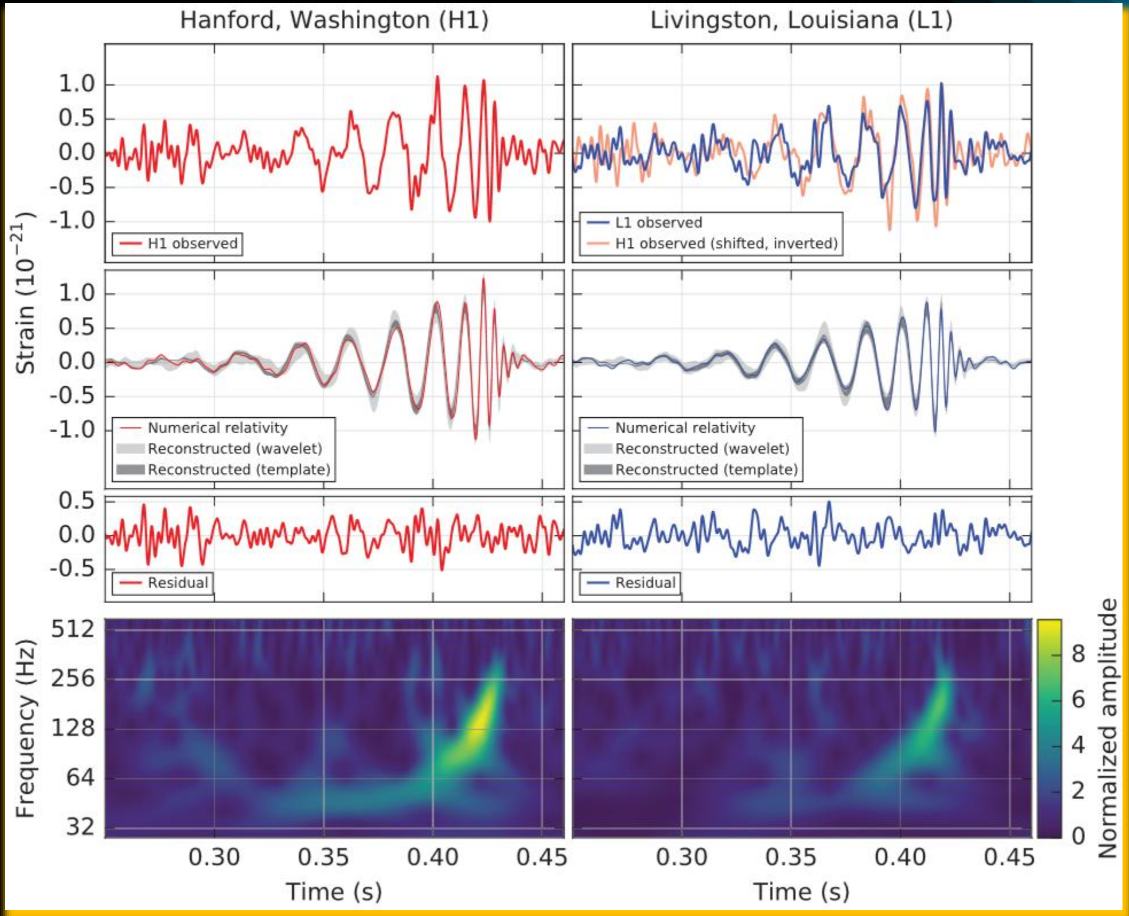
Dynamical scenarios

- **Asteroseismology**
  - Continuous gravitational waves
  - Merger
  - Oscillation after events
  - Quasi-Periodic Oscillations (X-ray)
- ***f*-mode oscillations**
  - Quadrupole
  - kHz
  - Ringdown after mergers
  - Einstein Telescope, Cosmic Explorer
  - Needs full general relativistic treatment



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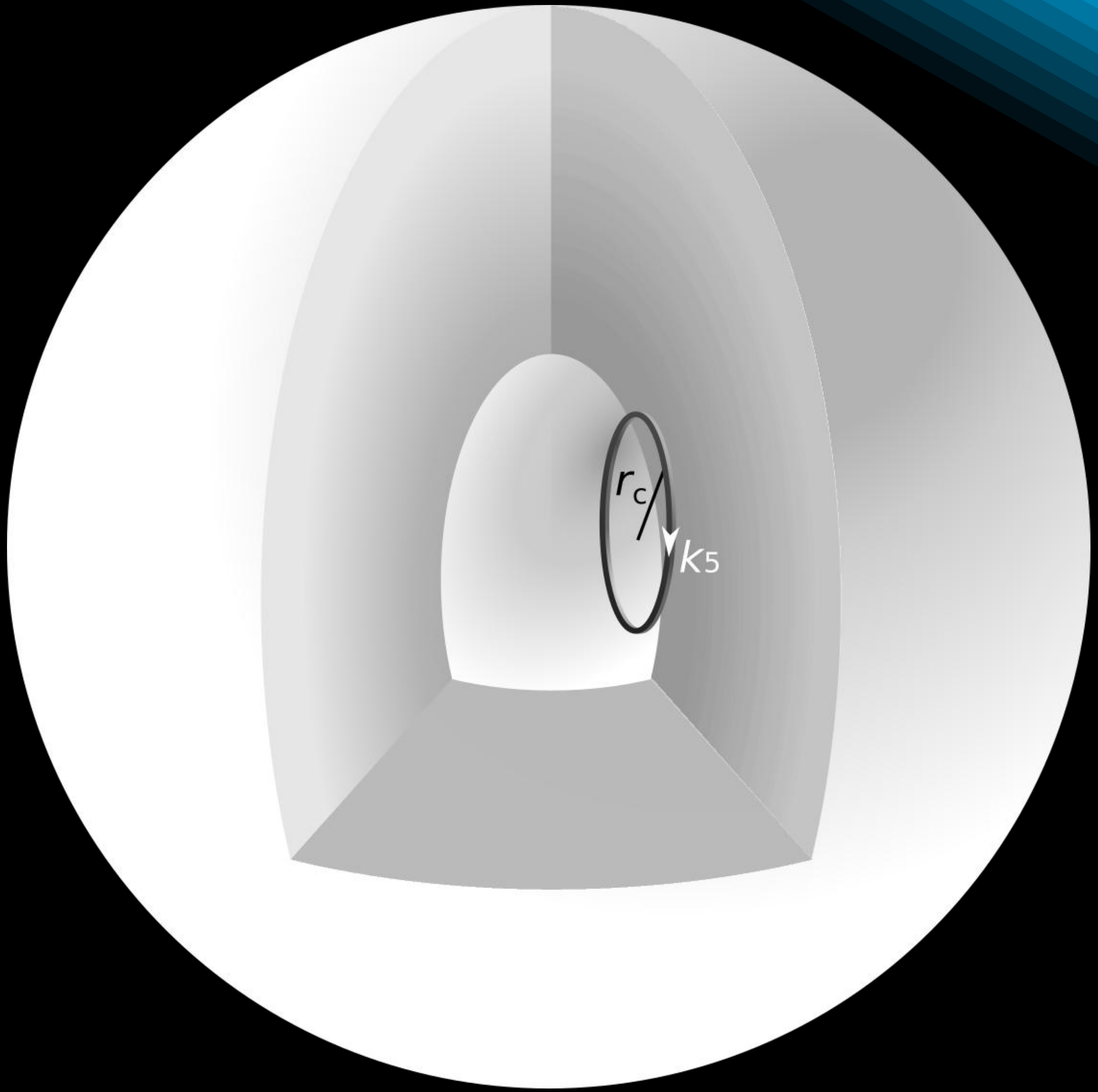
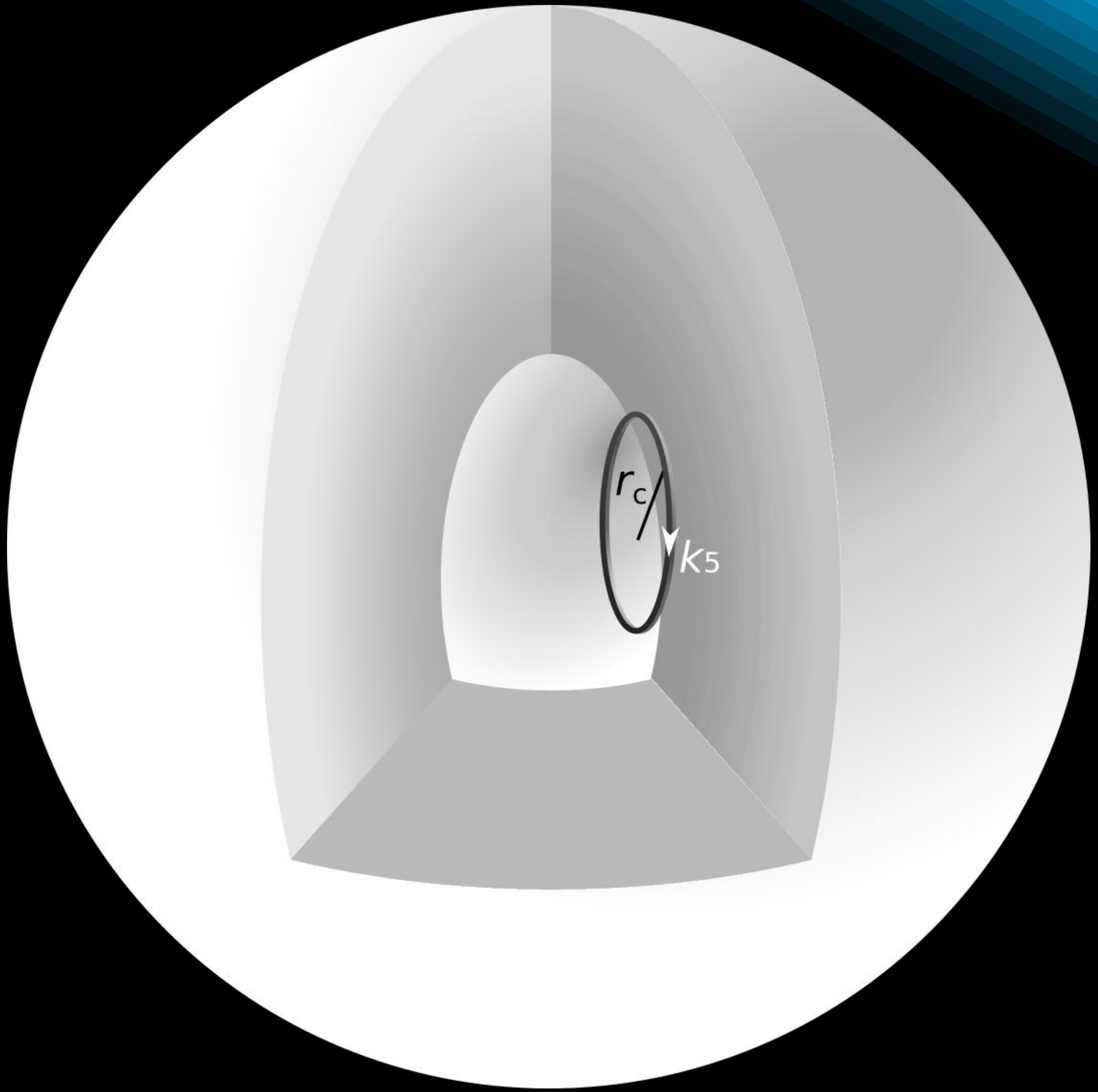
Summary

- **Extra dimensions**
  - Kaluza–Klein theory



Summary

- **Extra dimensions**
  - Kaluza–Klein theory
- **E**xtended **U**ncertainty **P**rinciple
  - Taking into account curvature in particle dynamics



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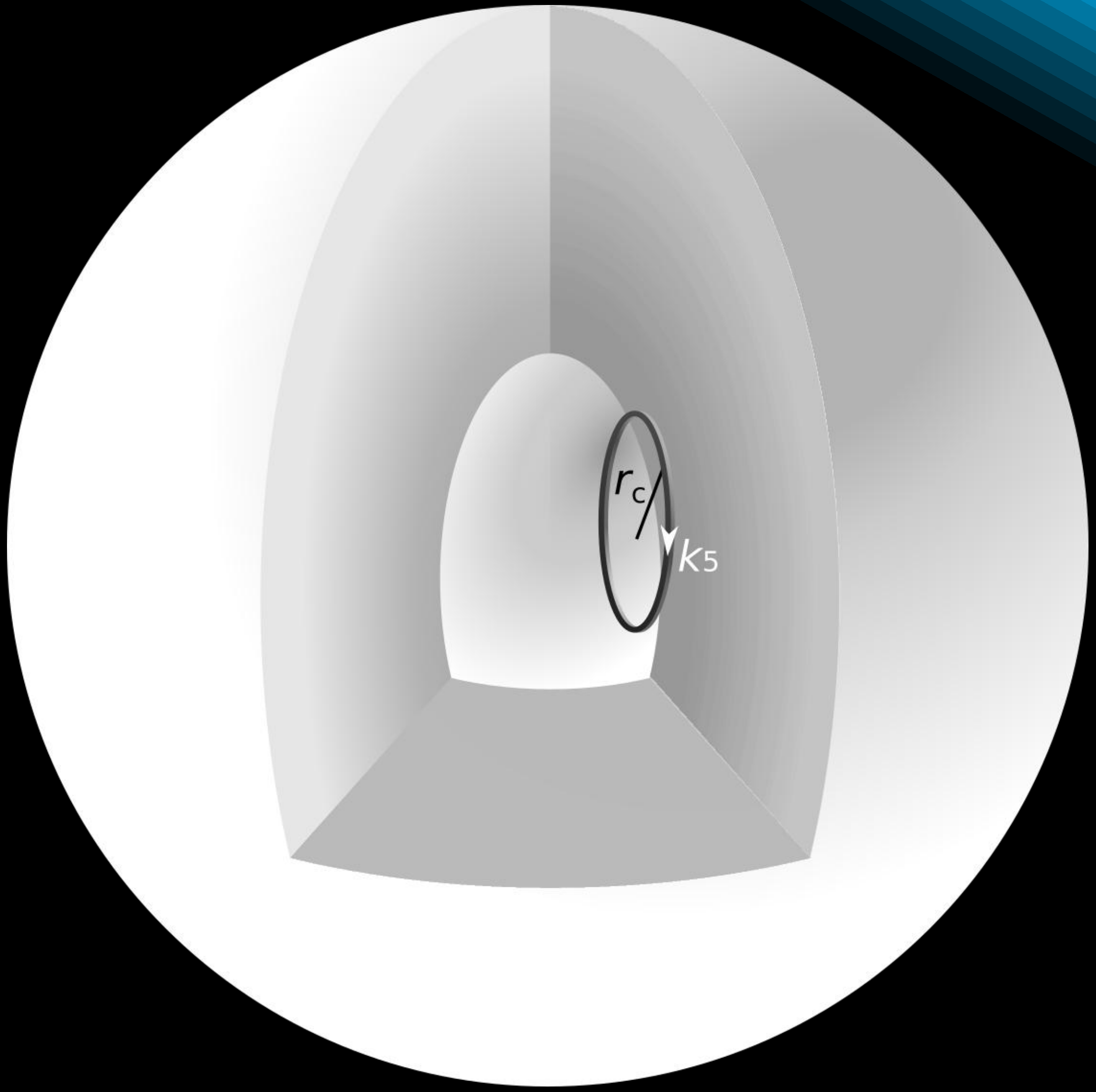
  - Taking into account curvature in particle dynamics
- **Equation of state**
  - Modelling static neutron stars
    - Computationally heavy
  - Dynamical simulations
    - Computationally heavier



Thank you for your attention!

HUN-REN KMP-2025-I/45
ERASMUS+ Short-term doctoral mobility
BridgeQG (CA23130) COST Action's Short Term Scientific Missions program
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