Prototypes of HVDC connections in a Kuramoto-like power-grid system

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Outline

1. Motivation
2. Model and prototypes
3. GPU implementation
4. Results
5. Conclusion

Main message

Question

Can a small number of HVDC-like, converter-controlled links improve synchronization and reduce cascade sizes in a large AC grid model?

Answer in one sentence

Yes for phase synchronization and cascades, but with a cost: DC segmentation improves local phase coherence while increasing frequency dispersion, and adaptive controls can make relaxation much slower.

The goal is a non-perturbative dynamical test of topology-driven synchronization and failure propagation, not a detailed protection-system model.

Why HVDC-like links are interesting now

Changing grids

- renewable generation is geographically variable
- inverter-based devices reduce classical rotating inertia
- large inter-area transfers become more important

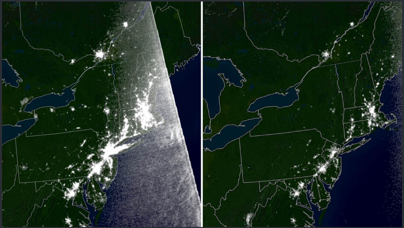
Control opportunity

- HVDC transfers power without enforcing AC phase synchrony
- converter controls can react to frequency information
- this suggests a controllable “firebreak” mechanism

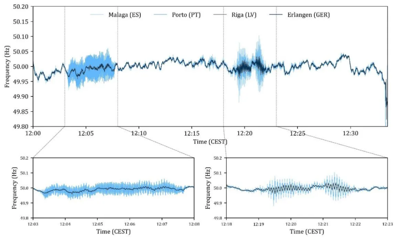
Talk focus

When is controlled separation stabilizing, and when does it create a new trade-off?

Blackouts as motivation, not calibration



2003 Northeast blackout: large-scale cascade propagation.



2025 Iberian event: inter-area oscillations before the major generation loss.

Need for investigation

Recent outages motivate stability studies and synchronization investigations via dynamical simulations.

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Base model: second-order Kuramoto / swing equation

$$\ddot{\theta}_i + \alpha\dot{\theta}_i = P_i + K\sum_j W_{ij}\sin(\theta_j - \theta_i)$$

Dynamical variables

- $\theta_i(t)$: phase
- $\dot{\theta}_i(t)$: frequency
- P_i: quenched source/load disorder

Network

EU2016 high-voltage graph: $N = 13\,478$, $E = 18\,393$, weighted by voltage/susceptance assumptions, with spectral dimension $d_s \simeq 1.5$.

How HVDC enters the dynamics

$$\ddot{\theta}_i + \alpha\dot{\theta}_i = P_i + K \sum_{j\in AC(i)} W_{ij} \sin(\theta_j - \theta_i) + \sum_{j\in DC(i)} D_{ij}\, f\left(\dot{\theta}_j - \dot{\theta}_i\right)$$

AC line

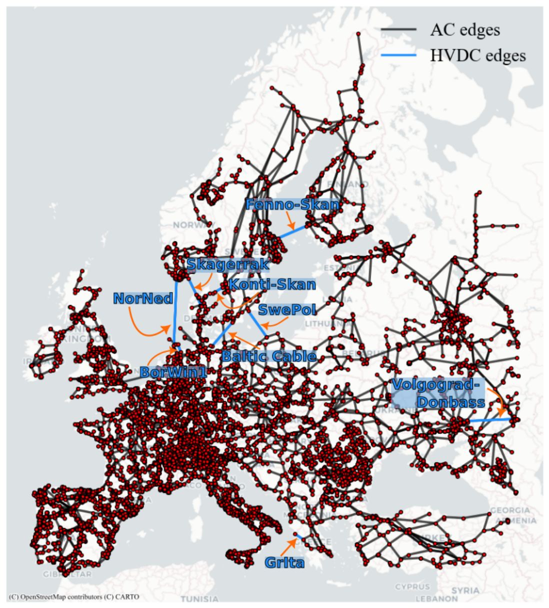
Power flow is tied to phase difference: $\sin(\Delta\theta)$.

DC line

Power is controlled by a converter rule using frequency difference $f(\Delta\omega)$ and capacity scale D_{ij}.

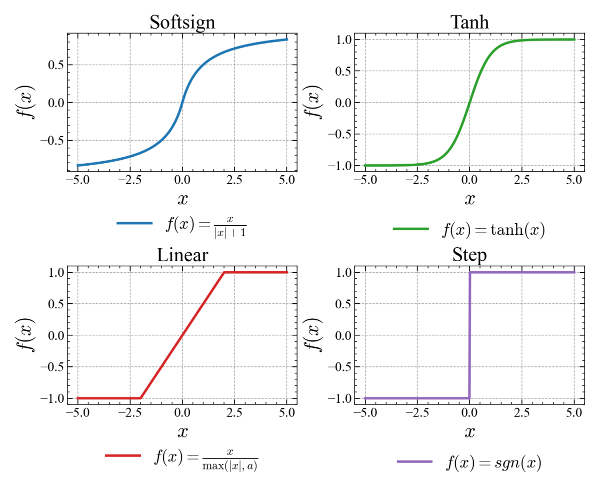
Static HVDC is modelled as constant source/sink injections at the endpoints.

Network and computational setting



Prototype activation functions

Prototype	Rule	Interpretation
Static HVDC	fixed $P_{d,i} = -P_{d,j}$	constant scheduled transfer; pure segmentation effect
Linear / dead-band	$f(x) = \mathrm{clip}(x/a, -1, 1)$	simple frequency-control proxy with tunable threshold
Smooth nonlinear	$f(x) = \tanh x$ or $x/(1 + \lvert x\rvert)$	bounded adaptive response without a hard jump
Threshold-like	$f(x) = \mathrm{sgn}(x)$	most sensitive response; discontinuous limiting case



Parameter intuition

Narrower linear intervals approach threshold control: better cascade reduction in the simulations, but harder numerical and dynamical relaxation.

What we measure and how

Synchronization

$$R(t) = \left\langle \left| \frac{1}{N} \sum_j e^{i\theta_j(t)} \right| \right\rangle,$$

$$\Omega(t) = \left\langle \frac{1}{N} \sum_j \left(\omega_j(t) - \bar{\omega}(t) \right)^2 \right\rangle.$$

Cascade test

After thermalization, remove one random AC edge. Then remove overloaded AC links with

$$F_{ij} = |\sin(\theta_j - \theta_i)| > T.$$

Report $\langle N_f \rangle$.

Numerics

Many quenched realizations, long transients, and post-perturbation cascades motivate GPU-parallel integration.

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GPU implementation choices

JAX / Diffrax

- very fast after compilation
- natural batching via `vmap`
- awkward dynamic topology changes

PyTorch / torchdiffeq

- flexible prototyping
- easy debugging
- GPU use depends strongly on implementation

PyCUDA

- more kernel and memory control
- good for custom kernels
- less solver infrastructure

Practical compromise

Use fixed-size arrays plus masks for topology changes where possible; complement GPU runs with CPU adaptive integration for checks.

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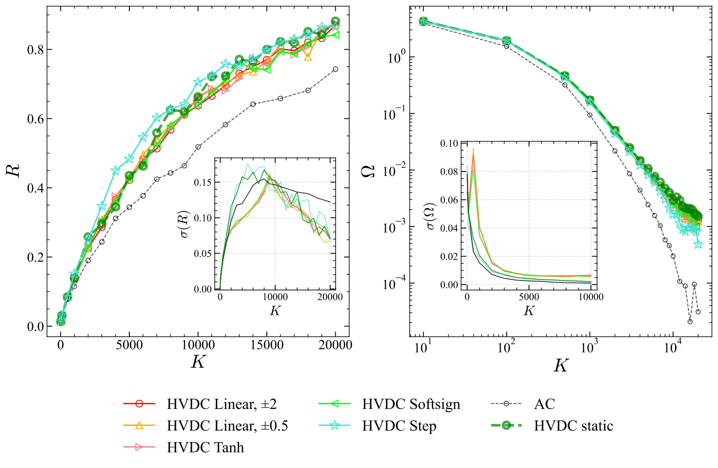
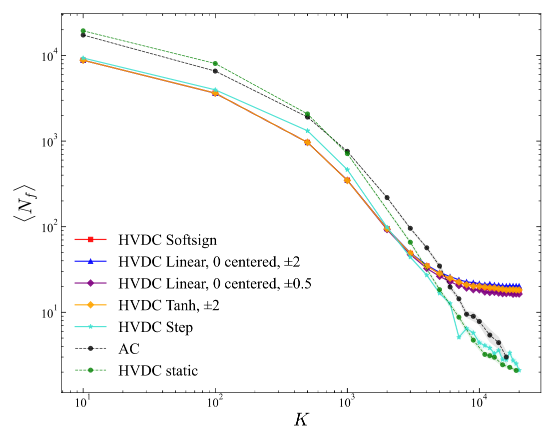
Synchronization vs. frequency spread



HVDC variants increase phase order *R*, frequency dispersion Ω becomes larger. DC edges transmit power without AC phase/frequency information.

Cascade sizes are reduced in the useful regimes



Choosing the static modelling or $sign(x)$ as activation function, results in a stabilizing, cascade-reducing effect for all investigated K values. Other methods display a crossover behaviour.

Stabilization is not free

What improves

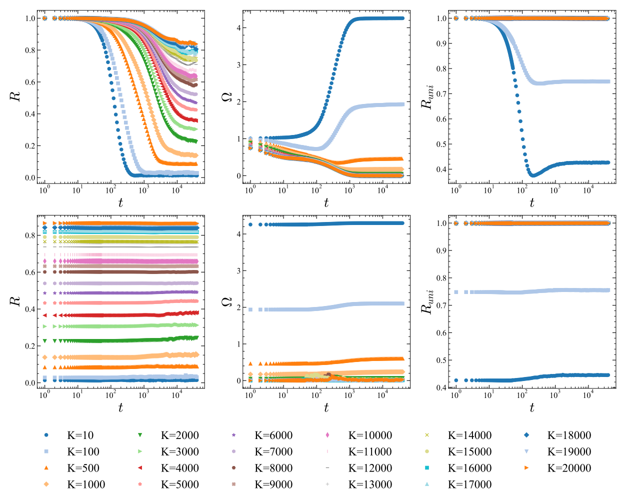
- higher phase order R
- smaller cascades $\langle N_f \rangle$
- controlled separation of weakly coupled regions

What gets worse

- larger frequency dispersion Ω
- possible deterioration outside optimal regimes
- adaptive HVDC can increase relaxation times

One has to strike a deal between "how much does it stabilize?" and "how fast does it settle?"

Long transients in adaptive controls



Adaptive smoothing can reduce abrupt control action, but it may also create very slow relaxation before the cascade test is meaningful.

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Where does this leave us?

Why segmentation helps R

HVDC replacement splits the AC synchronization problem into smaller effective subgraphs, which can display higher phase order at the same coupling K.

Why Ω worsens

The subgraphs no longer exchange phase/frequency information through those lines, so inter-area frequency differences can persist longer.

Scope

The model captures nonlinear phase/frequency dynamics and qualitative cascade propagation, but not voltage stability, reactive-power control, detailed converters, or real protection schemes.

Take-home messages and outlook

1. **Small HVDC changes can have large dynamical effects** because they alter AC synchronization topology.
2. **Phase synchronization and frequency entrainment respond differently**; improving one can worsen the other.
3. **Threshold-like controls reduce cascades most strongly**, but can create very slow relaxation.
4. **Next steps:** stochastic disturbances, measured frequency distributions, richer converter/control models, and HVDC placement optimization.

Closing line

HVDC links are not just transmission assets; in a nonlinear synchronization model they are control interfaces that reshape the graph itself.

Selected references

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- B. Hartmann, G. Ódor, K. Benedek, I. Papp, *Studying power-grid synchronization with incremental refinement of model heterogeneity*, arXiv:2409.02758; Chaos 35, 013138 (2025).
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