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# Cognitive algebra: context, concepts and the knowledge space

**GPU days 2026**

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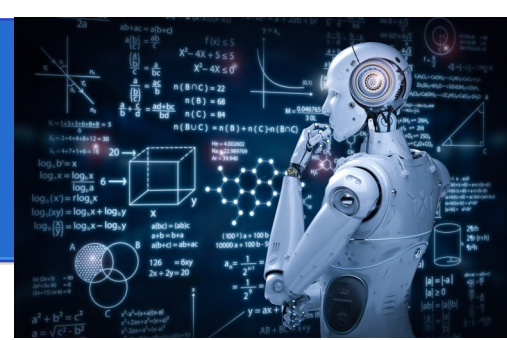
# Introduction: successes of AI

## We live in the era of AI...

- multimodal systems
  - ➔ text, code, image, speech, video recognition and generation
  - ➔ ChatGPT, Gemini, Deepseek, Grok, Claude, ...
- specific systems
  - ➔ games, autonomous cars, recommendation systems, protein structure, weather forecasting, medical diagnostics
- it is to be explained why NOT to use AI...
- it often matches or exceeds human-level performance (language models, games, medical diagnostics)



# Introduction: AI challenges



**How far can we go? → can a machine be smarter than its creator?**

## **traditional algorithms**

- we understand the **problem**, we write the code, the machine solves it
- machine is not smarter (only faster)

## **approach of AI**

- we understand **understanding**, write an understanding code
- machine understands and solves the problem
- the machine can be smarter in problem solving
- but: does not understand understanding better than we do!

# Introduction: AI challenges



**How far can we go?**

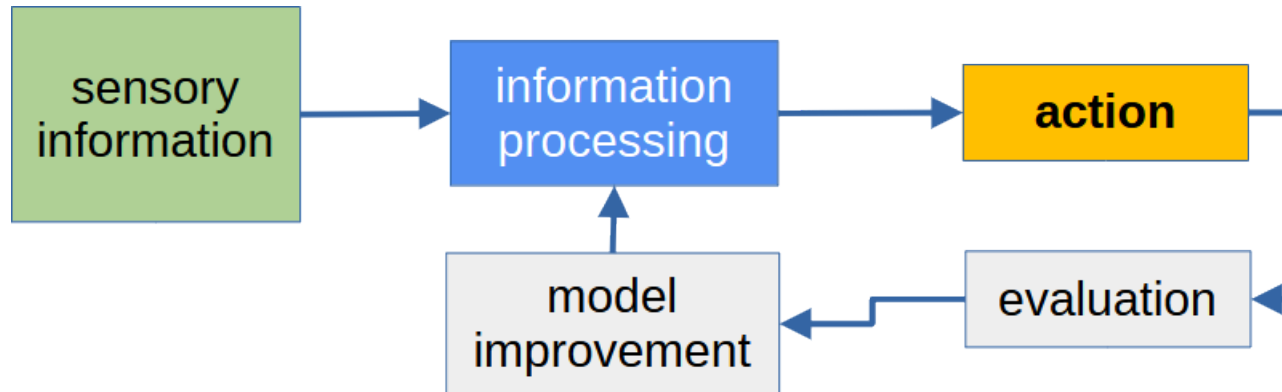
**answer:** depends how well do we understand **understanding!**

**actually:** what is our current picture of understanding?

# Correct picture of understanding

## Cognitive model of the present-day AI systems

- feed forward action focused system
- action can be returning the class, or the next token, or a real action (in case of robots)



# Correct picture of understanding

## **Mental model of the present-day AI systems → consequences**

- action directly comes from the input (including randomization)
- no self-verification, no chance to control the quality of the answer
- lacks awareness of its knowledge boundaries (doesn't know what it doesn't know, or what it does know → importance of the good question/prompt)
- intuitive, unconscious, slow to learn, fast to act
- in human psychology: System 1
- **mathematically**: function fitting

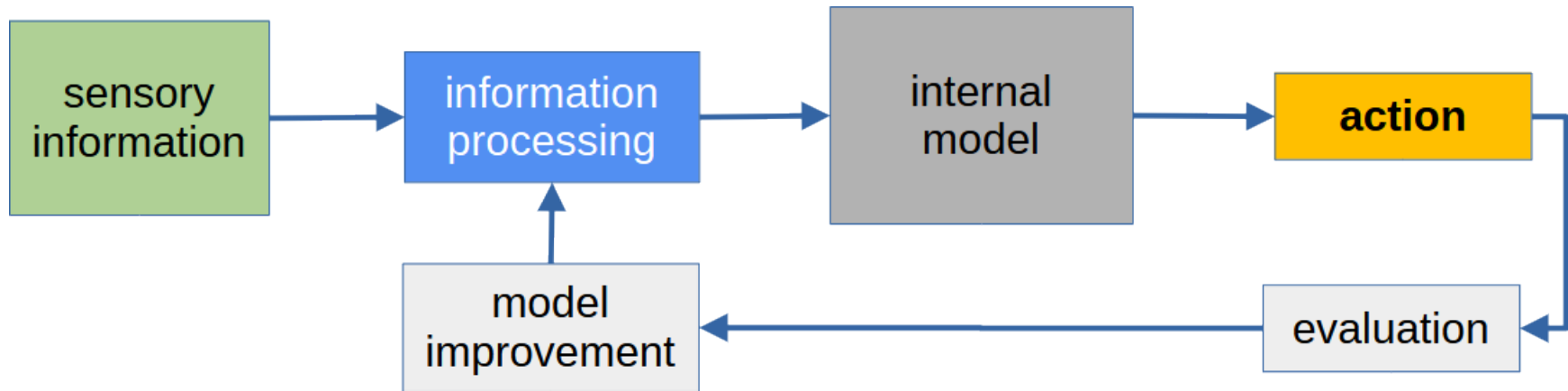
# Correct picture of understanding

## Is it the adequate picture of the understanding?

- **problem:** too many types of situation can happen, we can not work out the answer for all of them
  - not enough memory to store
  - not enough data to learn
- we need a mental model that reacts to unknown situations reasonably
- this model must **represent the** world (at least the relevant part of it)!
- System II, representation learning, ...

# Correct picture of understanding

## Cognitive model of System II



this is not a function fitting problem!

→ **what is the mathematics behind System II and world representation?**

# Math behind world representation

## Use a physical approach

→ define Universe ( $\Omega$ ), observer, space and time as in physics

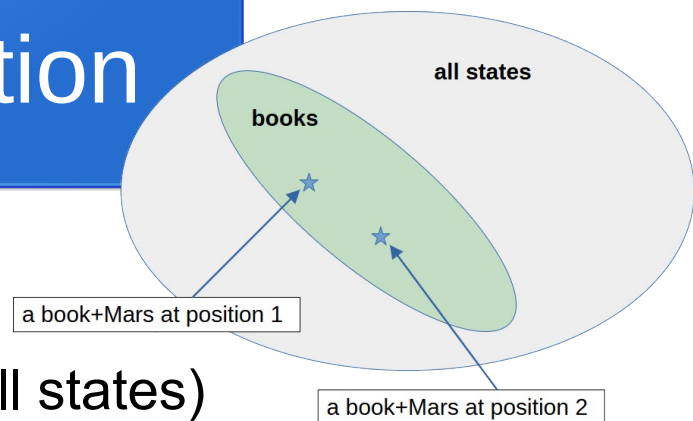
- **states**: the possible configurations of the Universe on a spacial instance, contains all information to be able to predict the future
- **observed world** is smaller than the Universe, a full description is not possible/necessary
  - a single observer has finite observation/processing capabilities
  - many details don't play role for the agent's purpose (e.g maintain life)
  - example: to describe the fate of a book, the position of Mars is irrelevant

**objects/concepts are collection of states**  
(differing in irrelevant details)

# Math behind world representation

**Classes/concepts/objects**  $\rightarrow O \subset \Omega$

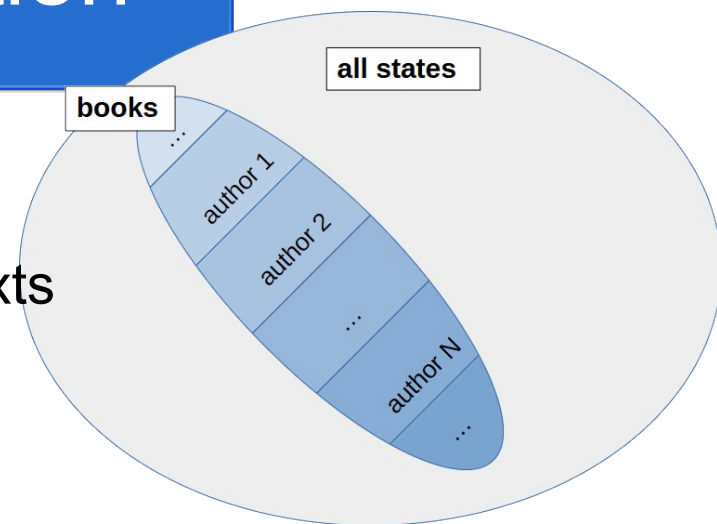
- classes/concepts are subsets of the Universe (all states)
- collects states differing only in irrelevant details
- examples:
  - books: collection of states where Mars is in different position, or the exact matter content (on atomic level) is different
  - red pixel: subset of images where the color of the given pixel is red



# Math behind world representation

## Contexts:

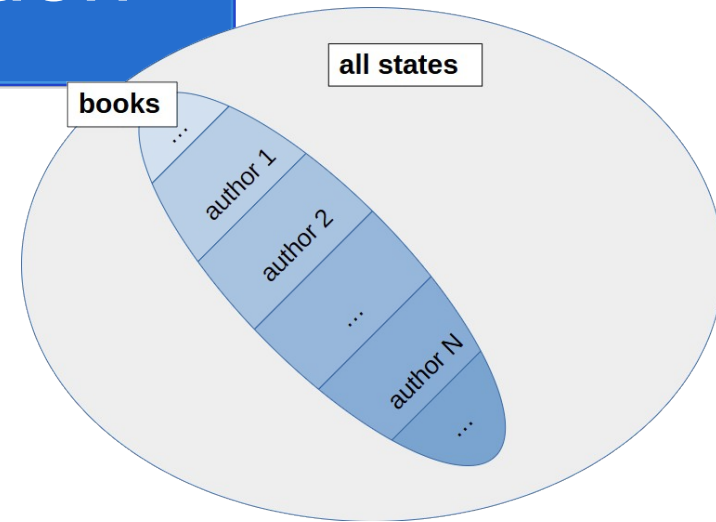
- we can speak about a concept in different contexts
- example
  - books from the point of view of a book shop, a collector, a reader or a mover
  - dogs from the point of view of their owner, or their breed
- all elements in a context are concepts (classes)
- definition context is a **partition of the underlying class**  $C = P(O)$



# Math behind world representation

## Contexts and physics

- in real world we never treat states, always use classes (objects)
- no ultimate description of the world
- the way we characterize the objects of the study: fix the **context**  
e.g.: an object described by its atoms or by its mechanical properties
- in elementary physics context usually is fixed by the **scale**
  - not true in a more complicated system)
  - context change → renormalization



# Math behind world representation

## Operation on contexts

- **coarsening**: all classes of a coarser concept is a union of classes of the original context
- **refining**: subdividing one/more classes of a context
- **examples**:
  - Jonathan, Gala, ... (apple varieties)  $\Leftrightarrow$  apple
  - individual images  $\Leftrightarrow$  cat/dog/other images
  - mammals, birds, reptiles, amphibians and fish  $\Leftrightarrow$  vertebrate

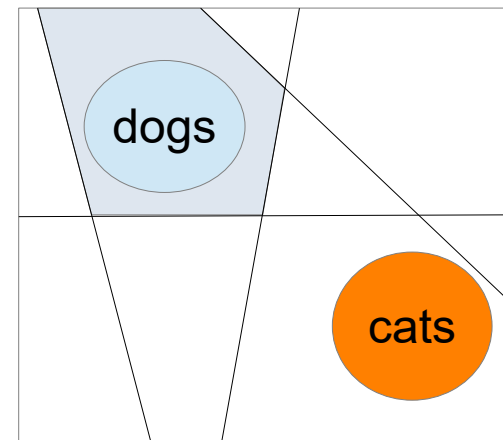
# Math behind world representation

## Representation of a complicated context

- **direct representation:** all classes of the context is treated separately
  - one-hot encoding, System 1
- **global coordinates:** several simple contexts, use common refinement

$$C = \bigwedge_i C_i, \text{ where } C_i \wedge C_j = \{o_i \cap o_j \mid o_i \in C_i, o_j \in C_j\}$$

- examples:  $V = \mathbb{R}^N$ , image as a set of pixels  
vector embedding in LLMs
- simple elementary contexts, exponential refinement



# Math behind world representation

## Representation of contexts

### ● local coordinates:

- start with a simple context (partition) of the ground set
- refine each classes independently
- results in a refinement tree
- **examples**: twenty question (barchoba), taxonomy

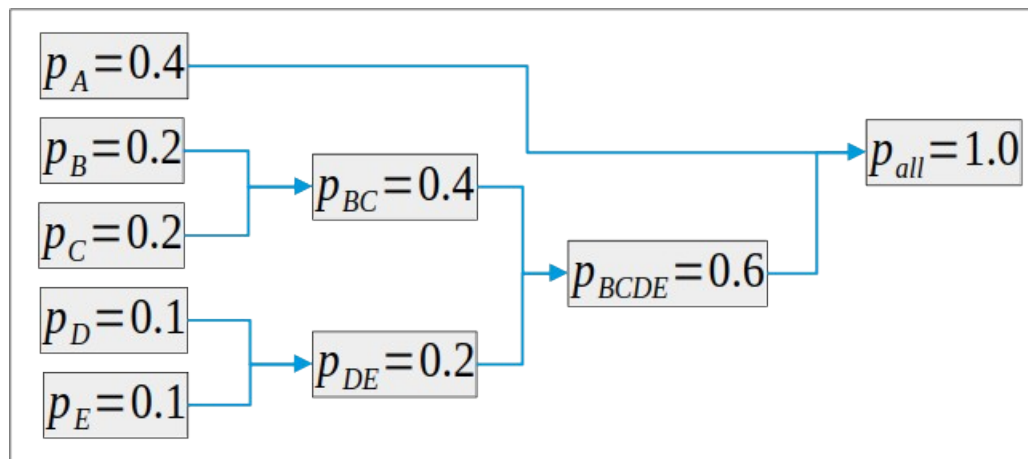
### ● optimal coding:

- information theory → Huffmann coding
- local coordinates implement it!

# Math behind world representation

## Huffmann coding

- assume we know the probabilities of each output
- combine the least probable events into a common (abstract) event



# Math behind world representation

## Numerical representation

- Universe is observed through measurements  $\Omega \rightarrow B$  ( $B$  finite set)
- **feature** of a context: constant on classes of the context (class-valued function)  
 $f: \Omega \rightarrow B, \omega, \omega' \in c \in C \Rightarrow f(\omega) = f(\omega')$ 
  - inverse image provides  $C' < C$  (induced context)
- other names
  - conserved quantities
  - measurable functions on  $(O, \sigma(C))$
  - selective/relevant quantity

# Math behind world representation

## Numerical task:

- collect data → measurements (numbers)
- single out the **context** (questions we want to answer)
  - for example sound recognition, image recognition
- try to find **context-valued functions** (conserved quantities)
  - find a sparse representation of the context (e.g. Fourier transform)
  - observe which coordinates are interdependent (causality, Hebb's rule)
  - use multiple simple elementary features (e.g. multi-linear → PCA, LLT)
- represent the world with the classes and its values

# Conclusion

- **current understanding of understanding:**
  - direct action focused (System 1) mental model
  - mathematically it is a function fitting
- **to represent reality (world model)**
  - representation focused (System 2) mental model
  - mathematically: find context-valued functions (conserved quantities) associated to a given context
- a lot of mental concepts (“thinking”, “self”) can appear only in the world model

The end

