

Modelling Neutron Stars in Scalar Tensor Theories

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Scalar-Tensor Theories: Beyond General Relativity

General Theory of Relativity

Gravity is described purely by metric $g^{\mu\nu}$

$$S_{GR} = \frac{1}{16\pi G} \int R \sqrt{-g} d^4x + S_{\text{matter}}$$

Scalar Tensor Theory

Gravity mediated by $g^{\mu\nu}$ + scalar field φ

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)] + S_{\text{matter}}(A^2(\varphi)g_{\mu\nu}, \varphi)$$

(Einstein Frame)

Motivation? General Relativity may not be the complete theory of gravity; extensions like STT introduce additional degrees of freedom.

Why a scalar field? Appears naturally in many fundamental theories.

Why neutron stars? Neutron stars probe the strong-gravity regime, where deviations from GR could become observable.

Scalar Tensor Theory:

Two ways to write the same theory

Jordan Frame

$$S = \frac{1}{16\pi} \int \left(\varphi \tilde{R} - \frac{\omega(\varphi)}{\varphi} (\tilde{\nabla}\varphi)^2 - V(\varphi) \right) \sqrt{-\tilde{g}} d^4x + S_m(\tilde{g}_{\mu\nu}, \chi)$$

- Physical frame.
- Test particles follow geodesics.
- Gravity is modified.
- Field equations look complicated.

Conformal Rescaling

$$\tilde{g}_{\mu\nu} = A^2(\varphi) g_{\mu\nu}$$

Einstein Frame

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)] + S_{\text{matter}}(A^2(\varphi) g_{\mu\nu}, \chi)$$

- Mathematical frame
- Gravity looks like clean GR
- Scalar field φ has its kinetic + potential term; Matter couples to the conformal scaling function.
- Used in your paper — simpler ODEs.

Neutron Stars In Scalar Tensor Theories With A Massive Scalar Field

- Einstein Frame Action:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)] + S_{\text{matter}}(A^2(\varphi)g_{\mu\nu}, \chi).$$

- Potential:

$$V(\varphi) = 2m_\varphi^2 \varphi^2$$

- Class of STT: Damour & Esposito-Farèse class of STT

$$\alpha(\varphi) = \frac{d \ln A(\varphi)}{d\varphi} \longrightarrow \alpha(\varphi) = \beta\varphi \Leftrightarrow A(\varphi) = \exp(1/2\varphi^2)$$

- Equation of State: APR (Akmal–Pandharipande–Ravenhall)

Why does Scalarization Occur?

Vary action with respect to φ :

$$\nabla_{\mu} \nabla^{\mu} \varphi = -4\pi G \alpha(\varphi) T + \frac{1}{4} \frac{dV}{d\varphi}$$

$$\square \delta\varphi = m_{\text{eff}}^2 \delta\varphi$$

where, $m_{\text{eff}}^2 = m_{\varphi}^2 + 4\pi G \beta(\epsilon - 3P), \beta < 0$

(Trivial Solution:GR)

$$\varphi = 0$$

(Perturbation)

$$\varphi = 0 + \delta\varphi$$

$$m_{\text{eff}}^2 > 0$$

$$m_{\text{eff}}^2 < 0, 4\pi G \beta(\epsilon - 3P) > m_{\varphi}^2$$

- $\delta\varphi$ oscillates and goes to zero.
- Stable \rightarrow GR

$$\delta\varphi \sim e^{\pm i\omega t}$$

$$\delta\varphi \sim e^{\pm \omega t}$$

- $\delta\varphi$ grows exponentially.
- Scalarization

$$4\pi G |\beta| (\epsilon - 3p) > m_{\varphi}^2 \quad \beta < 0 \quad \text{Tachyonic Instability Condition} \longrightarrow \text{Spontaneous!}$$

Massless vs Massive Scalar Field

Property	Massless	Massive
V(ϕ)	0	$2m_\phi^2\phi^2$
Exterior falloff	$1/r$	$e^{(-m_\phi)/r}$
Range	∞ (infinite)	$\lambda\phi = 1/m_\phi$ (finite)
Instability condition	$4\pi G \beta \varepsilon > 0$	$4\pi G \beta \varepsilon > m_\phi^2$
Which stars scalarize	above a critical compactness	only in a limited density window
Observational constraints	Binary pulsars + GW — tightly constrained	Only NS observations — large (β, m_ϕ) space

Why massive STT is physically richer

1. Massless STT is nearly ruled out. Binary pulsar observations constrain $|\beta|$ to sit just above 4.35 — an extremely narrow viable window. Most of the massless parameter space is already excluded.
2. Massive STT has a much larger viable parameter space
3. No scalar field emission detected in binary pulsars. In a binary system, if the scalar field had infinite range it would carry energy away and alter the orbital decay — this is not observed. A massive scalar field decays exponentially beyond Compton Wavelength and never reaches the companion star, so no scalar radiation is emitted. Consistent with observations.

Spacetime Metric and Reduced Einstein Frame Field Equations

$$ds^2 = -e^{2\nu(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) - 2\omega(r, \theta) r^2 \sin^2 \theta d\phi dt$$

Gravitational Mass

$$\frac{1}{r^2} \frac{d}{dr} \left[r \left(1 - e^{-2\Lambda} \right) \right] = 8\pi G \left[A^4(\varphi) \tilde{\epsilon} + e^{-2\Lambda} \left(\frac{d\varphi}{dr} \right)^2 + \frac{1}{2} V(\varphi) \right]$$

conformal Rescaling

Gravitational Potential

$$\frac{2}{r} e^{-2\Lambda} \frac{d\nu}{dr} - \frac{1}{r^2} \left(1 - e^{-2\Lambda} \right) = 8\pi G A^4(\varphi) \tilde{p} + e^{-2\Lambda} \left(\frac{d\varphi}{dr} \right)^2 - \frac{1}{2} V(\varphi)$$

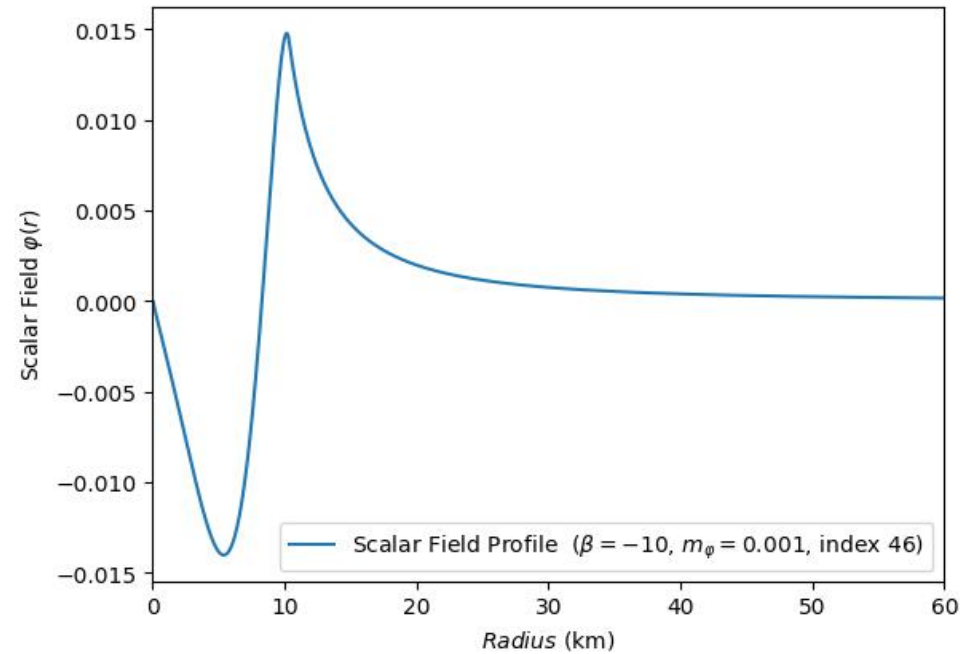
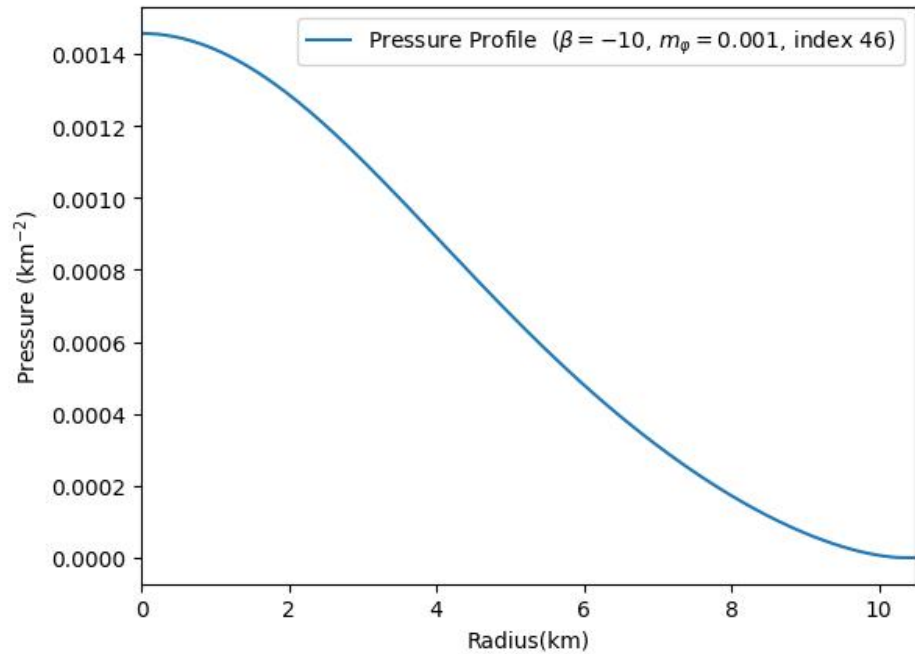
Scalar Field Equation

$$\frac{d^2\varphi}{dr^2} + \left(\frac{d\nu}{dr} - \frac{d\Lambda}{dr} + \frac{2}{r} \right) \frac{d\varphi}{dr} = 4\pi G \alpha(\varphi) A^4(\varphi) (\tilde{\epsilon} - 3\tilde{p}) e^{2\Lambda} + \frac{1}{4} \frac{dV(\varphi)}{d\varphi} e^{2\Lambda}$$

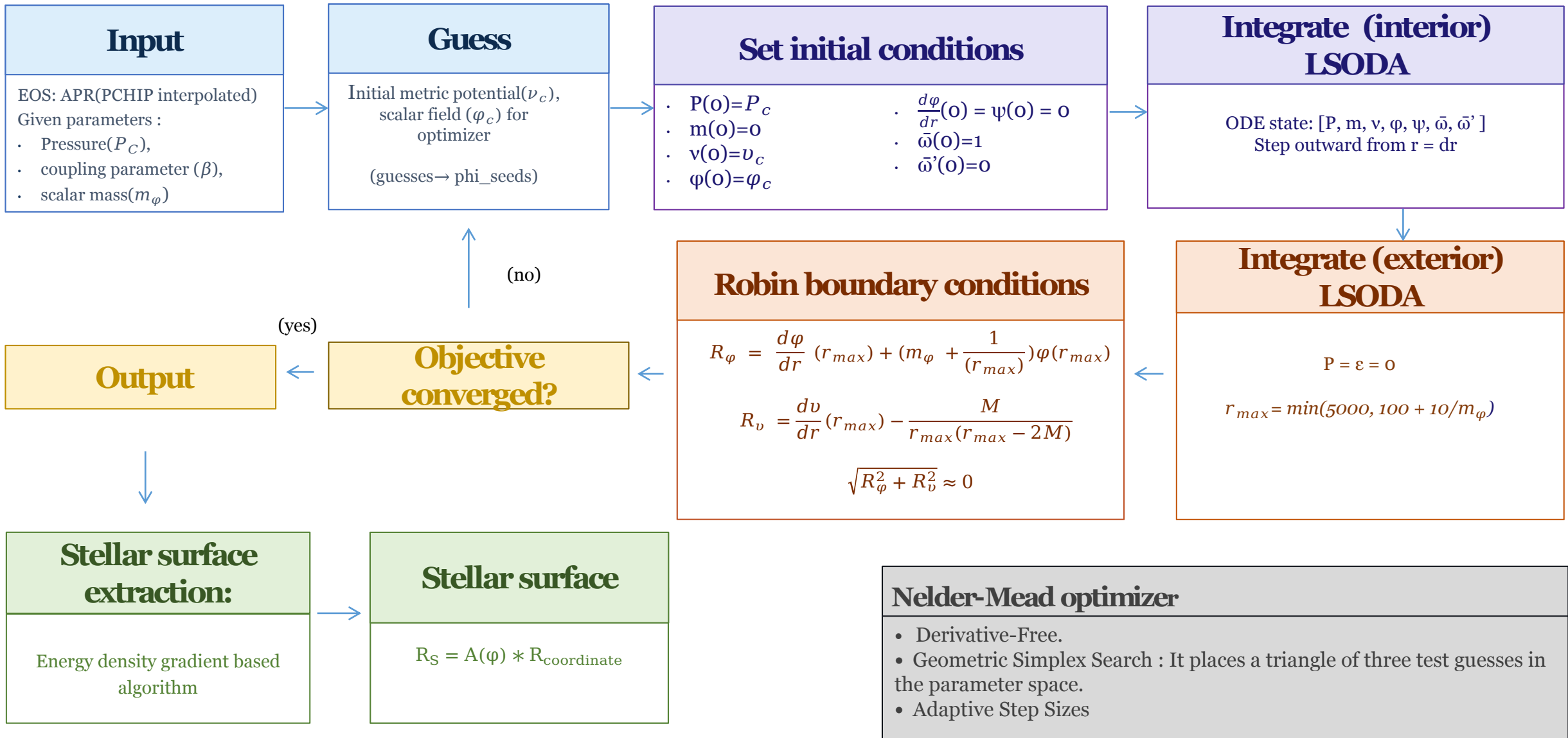
Hydrostatic Equilibrium

$$\frac{d\tilde{p}}{dr} = -(\tilde{\epsilon} + \tilde{p}) \left(\frac{d\nu}{dr} + \alpha(\varphi) \frac{d\varphi}{dr} \right)$$

Internal Radial Profile



Modelling: Shooting Method



Improved Stellar Surface Detection: Algorithm Overview

Radial Profiles: $\beta = -6$, $m_\phi = 0.001$

$r, P, m(r), v, \Lambda, A(\phi), \phi, d\phi/dr, n_b, \varepsilon$

Compute $d\varepsilon/dr$

energy-density gradient along r

Detect Near Surface Points

position of steepest negative gradient and near zero points ($d\varepsilon/dr < 0.01 \cdot \min(d\varepsilon/dr)$)

Cubic Zero-Crossing Fit

$f(r) = a_0 + a_1r + a_2r^2 + a_3r^3, f(R_{surf}) = 0$

Moving average + Spline interpolation

window of 3 points

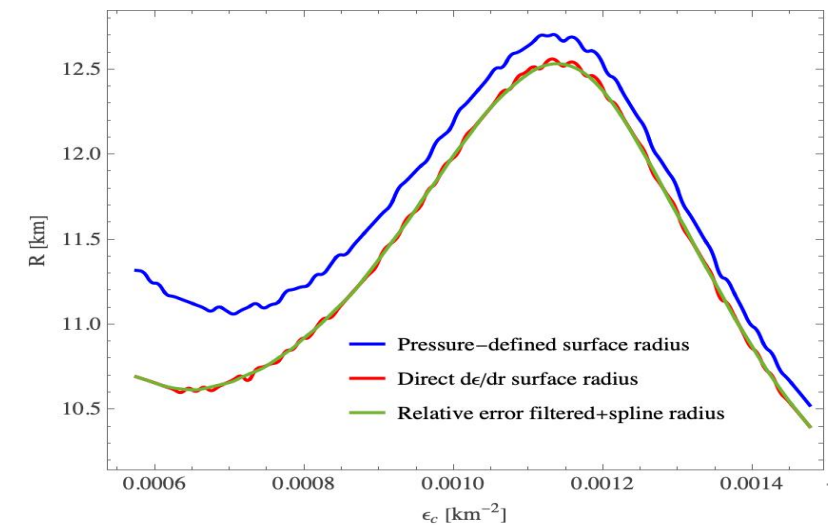
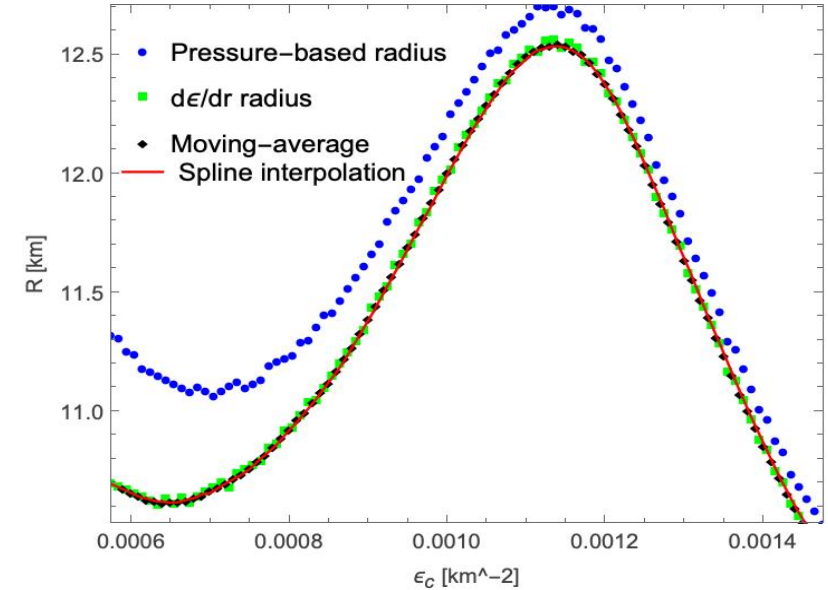
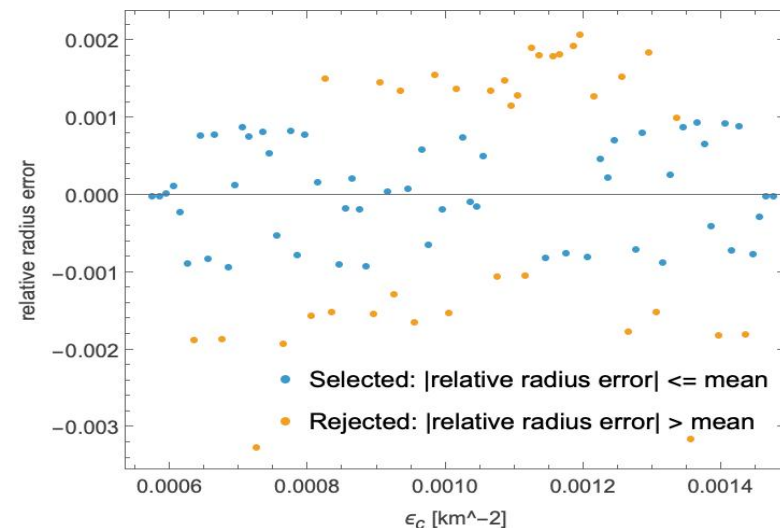
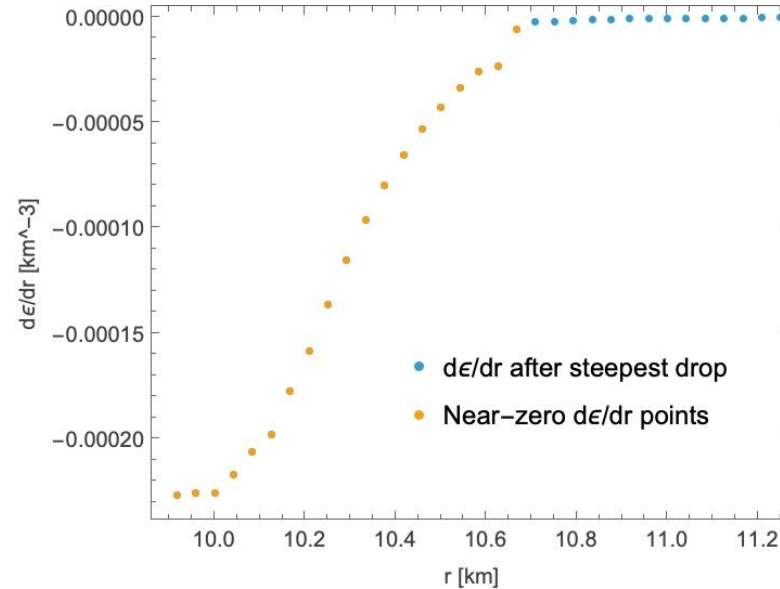
Selected Points

Relative error between the $d\varepsilon/dr$ based radius and the smoothed spline radius.

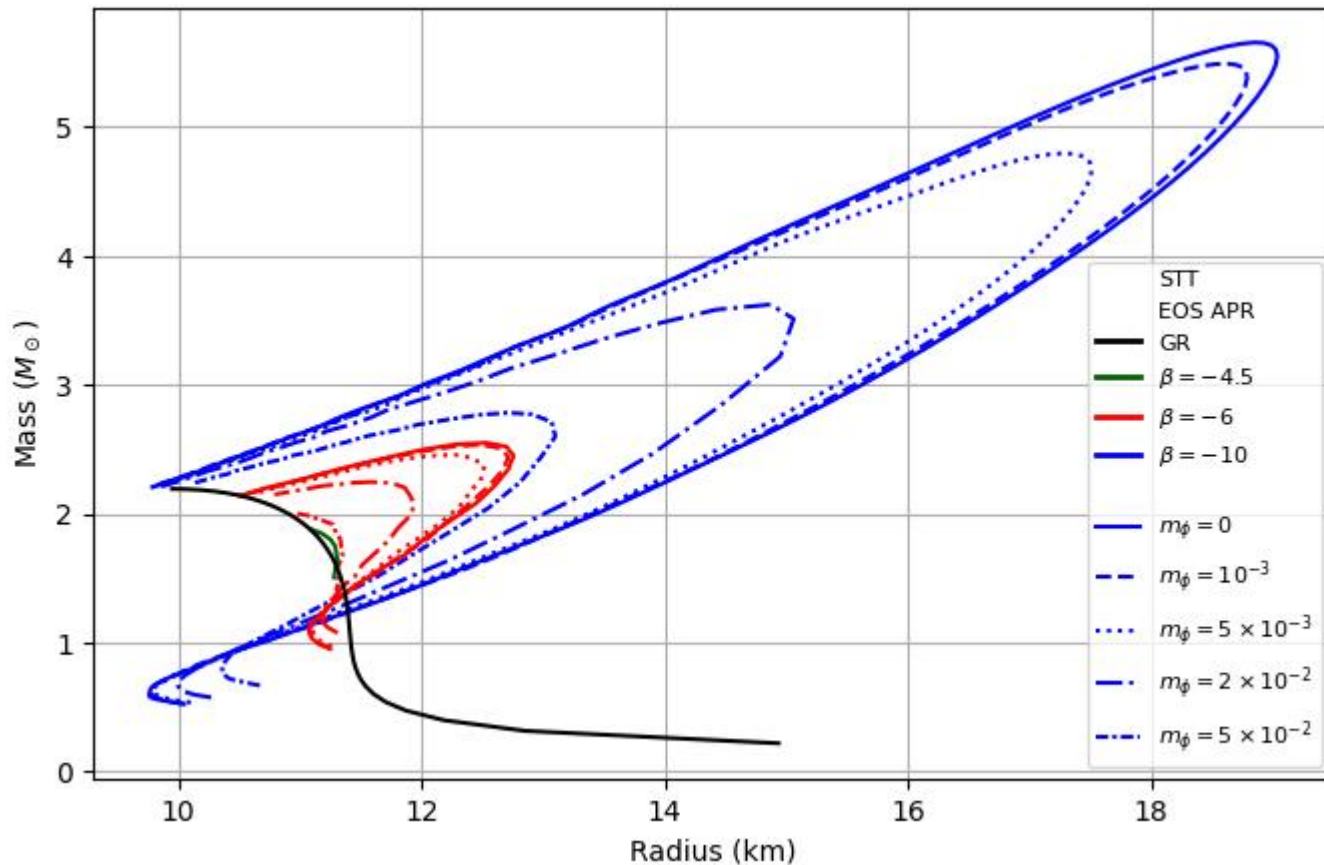
Smooth & Filter Radius Sequence

Spline of selected points

Diagnostic Plots

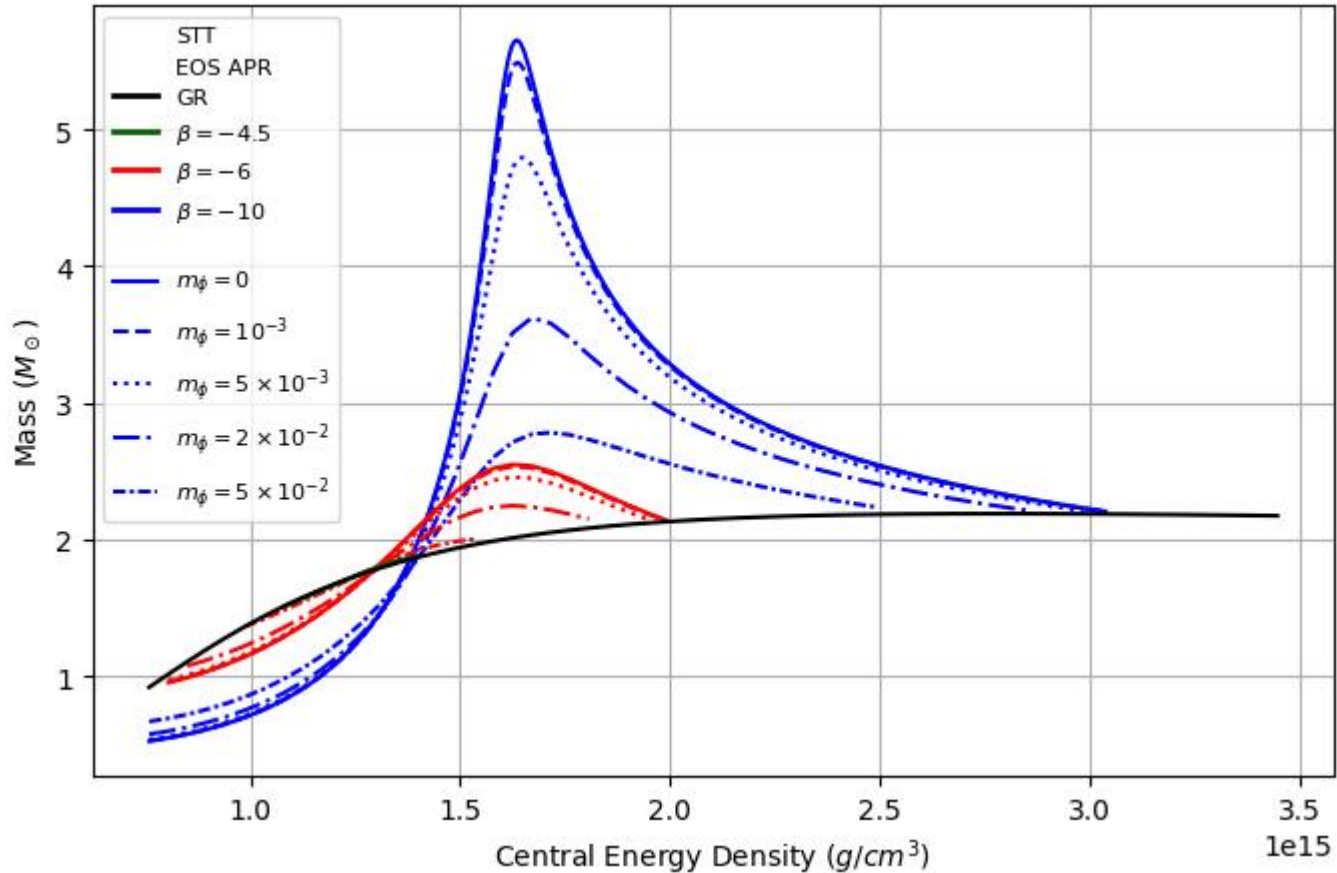


Mass vs Radius



- Increasing m_{ϕ} shrinks the loops toward GR.
- The loops represent the scalarized branch.
- Larger $|\beta|$ gives bigger loops.
- The scalar field surrounding the star contributes extra energy to the total gravitational mass.

Mass vs Central Energy Density



Low Density $4\pi|\beta|\varepsilon < m_\phi^2$

No scalarized solution \rightarrow GR

Critical Density $4\pi|\beta|\varepsilon = m_\phi^2$

Instability begins

Scalarized Branch $\varphi \neq 0$

Modified gravity

Maximum mass $\frac{dM}{d\rho_c} = 0$

Unstable Branch $\frac{dM}{d\rho_c} < 0$

Collapse to unstable stars

Rigid Slow Rotation: Moment of Inertia vs Mass

Hartle-Thorne approximation at first-order in the star's angular velocity

The first order captures only the effects of frame dragging; star remains a perfect sphere.

$$\frac{e^{\Phi-\Lambda}}{r^4} \frac{d}{dr} \left[e^{-(\Phi+\Lambda)} r^4 \frac{d\bar{\omega}(r)}{dr} \right] = 16\pi G A^4(\varphi) (\rho + p) \bar{\omega}(r)$$

where, $\bar{\omega} = \Omega - \omega$.

$\bar{\omega} \rightarrow$ The rotation speed of the stellar fluid relative to space.

$\omega \rightarrow$ The rotation speed of space itself (frame dragging).

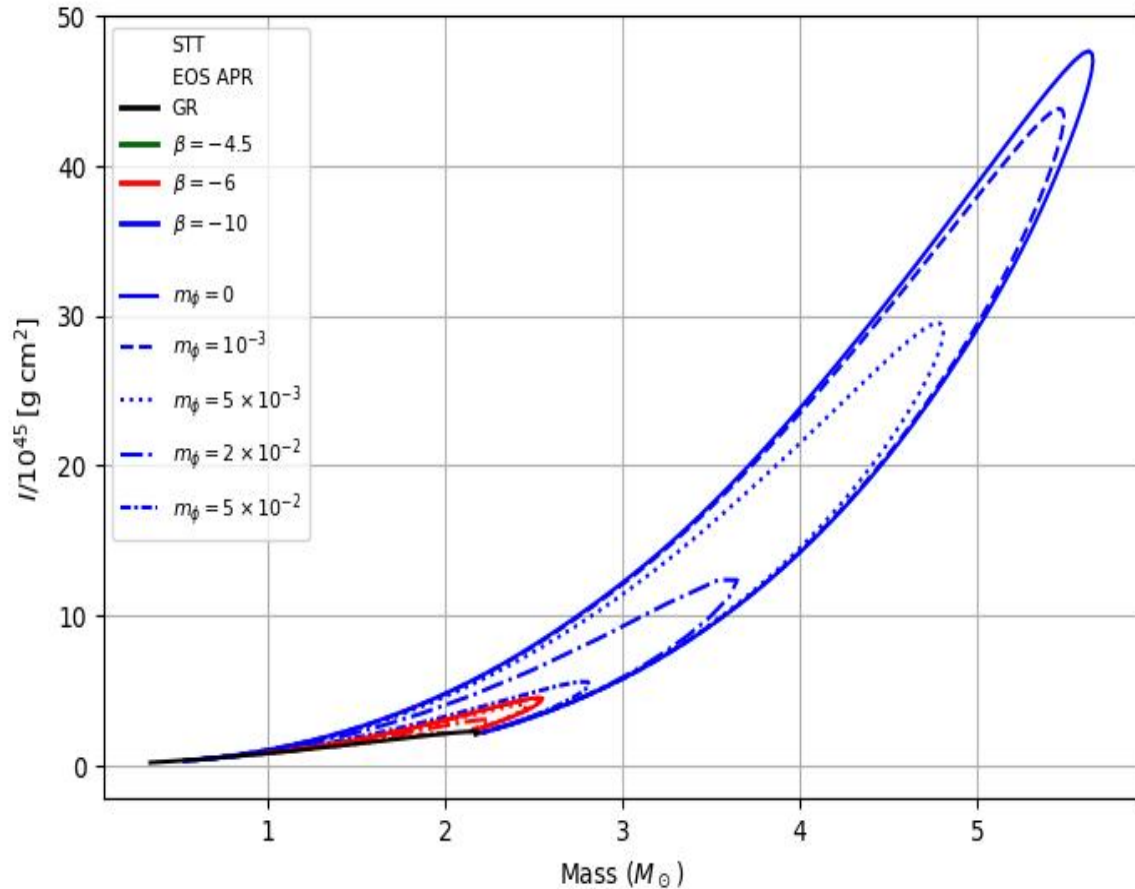
$\Omega \rightarrow$ The physical rotation speed of the neutron star.

The asymptotic solution at the boundary (r_{end}) is:

$$\bar{\omega}(r_{end}) = \Omega \quad \text{and} \quad \bar{\omega}'(r_{end}) = \frac{6J}{r^4}$$

$$\text{Moment of Inertia: } I = \frac{J}{\Omega} = \frac{r^4 \bar{\omega}'(r_{end})}{6 \bar{\omega}(r_{end})}$$

$J \rightarrow$ Angular momentum



Technical Challenges

THE TRIVIAL GR SOLUTION TRAP

Shooting Method Degeneracy

If the optimizer starts its search too close to the GR state, it will get trapped.

STIFF ODE

Interior-Exterior Stiffness

Different physical variables change at vastly different rates. LSODA dynamically monitors stiffness, automatically switching methods to maintain precision

SURFACE DETECTION

Pressure-Zero Ambiguity

Pressure \rightarrow 0 crossing occurs between grid cells, not at a sharp boundary. The code fits a cubic polynomial to locate the exact sub-grid surface.

MASSIVE SCALAR

The Exponentially Growing Mode in the Vacuum

Robin boundary condition that eliminates the growing mode, ensuring the scalar field is purely decaying.

SCALARIZATION THRESHOLD

Spontaneous Scalarization Filter

Some solutions are numerically indistinguishable from GR.

$$\varphi_c < \text{Threshold}(10^{-5} \text{ or } 10^{-6}) \\ \rightarrow \text{classified as GR}$$

RUNTIME

Multi-Dimensional Shooting Complexity

Computationally expensive and slow to converge

Conclusions

- The scalar field contributes extra energy and stiffens the star — scalarized stars can support significantly more mass than GR allows for the same equation of state.
- Massive scalar-tensor theories remain a viable alternative to GR for the strong-field regime.
- Our results successfully match established literature, but achieve higher numerical accuracy and stability by extracting the stellar surface at the sub-grid point.
- The next step is to extend this framework to compute Love numbers and tidal deformabilities

References

1. Yazadjiev, S.S., Doneva, D.D. and Popchev, D., 2016. Slowly rotating neutron stars in scalar-tensor theories with a massive scalar field. *Physical Review D*, 93(8), p.084038.
2. Hu, Z., Gao, Y., Xu, R. and Shao, L., 2021. Scalarized neutron stars in massive scalar-tensor gravity: X-ray pulsars and tidal deformability. *Physical Review D*, 104(10), p.104014.
3. Pani, P. and Berti, E., 2014. Slowly rotating neutron stars in scalar-tensor theories. *Physical Review D*, 90(2), p.024025.

Thank you for your attention